Recursive Computation of the Derivatives

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INTRODUCTION

The iterative solution of the inverse Time Harmonic Field Electric Logging problem is based on the repetitive use of the forward modeling algorithm. Information provided by the value and deriva-tives of the measurement given by the current model is used to determine the parameters of the model to be used in the following iteration. So, local knowledge of the derivatives of the measurement function is required at each iteration. Although numerical approximation of the derivatives can be easily performed, the possibility of computing them analytically would not only improve the convergence of the algorithm, but it also would improve its efficiency in terms of computational time.

This report describes the recursive procedure developed for the computation of the derivatives required for the implementation of the inverse problem. Due to the complexity of the measurement function, only its first derivatives are actually computed and a linear inversion approximation is considered.

DIFFERENTIATION OF THE MEASUREMENT FUNCTION

As it can be seen from [1], the solution of the Time Harmonic Field Electric Logging problem, which constitutes the logging tool measurement, is a very complex function of a large number of variables. Since the inverse problem is basically concerned with the determination of the earthen formation parameters, the derivatives with respect to some of those parameters (specifically the conductivities) are the ones we are interested in. A more detailed discussion on why the zones' radii are not been considered for the inverse modeling is presented in [2].

As it is described in [1], the computation of the measurement is performed in two clearly defined steps. In the first step, the electromagnetic responses for the current elements are computed. This responses are represented by the quantities denoted as ΔR's. In the second step, the method of moments is used to approximate the logging tool measurement. This is done by linearly combining the current element responses. As the second step performs a linear combination of the ΔR's, then the derivatives of the measurement can be certainly computed by performing the same linear combination with the derivatives of the ΔR's. For this reason we will be only considering the derivatives of the ΔR's.

Also from [1], it can be seen that the values of the ΔR's are defined (after a small change of notation) by the following integral expression:

$$\Delta R(z) = \frac{-2}{r_0\, h\, \pi^2} \int_{-\infty}^{\infty} Z_1(\lambda)\, \frac{\mathrm{Sin}^3(\lambda\, h/2)}{\lambda^3}\, e^{-j\lambda z}\, d\lambda \tag{1}$$

where:

$$Z_1(\lambda) = -\frac{\beta_1}{\sigma_1}\, \frac{K_0(\beta_1 r_0) + \Gamma_1\, I_0(\beta_1 r_0)}{K_0'(\beta_1 r_0) + \Gamma_1\, I_0'(\beta_1 r_0)}, \tag{2}$$

$$\beta_1 = \beta_1(\lambda) = \sqrt[2]{\lambda^2 + j\, \omega\, \mu\, \sigma_1}, \tag{3}$$

r_0 is the radius of the current element, h is the segment length, ω is the angular frequency of operation, μ is the magnetic permeability, σ_1 is the electric conductivity of zone 1, Γ_1 is the reflection coefficient of zone 1 (which contains the information related to all the zones in the formation), and I_0 and K_0 are the zero order Modified Bessel functions of first and second kind.

The definition given in (2) is actually the wave impedance evaluated at the current element's surface, r_0; and it is indeed a function of all the parameters of the earthen formation (radii and conductivities) which are contained in the value of the reflection coefficient Γ_1.

Again, because of the linearity of integration, the derivatives of ΔR(z) with respect to the conductivities can be computed as follows:

$$\frac{\partial\,\Delta R(z)}{\partial\,\sigma_n} = \frac{-2}{r_0\,h\,\pi^2}\int_{-\infty}^{\infty}\frac{\partial\,Z_1(\lambda)}{\partial\,\sigma_n}\;\frac{\mathrm{Sin}^3(\lambda h/2)}{\lambda^3}\;e^{-j\lambda z}\,d\lambda \qquad \text{for } 1 \le n \le N \tag{4}$$

where N is the total number of zones in the formation and σ_n is the electric conductivity of zone n. For simplicity in notation, $Z_1(\lambda)$ is going to be denoted as Z_1 from now on.

As it can be seen from (4), only differentiation of Z_1 is required; and the integral in (4) can be numerically approximated by using the same methodology developed in [3] for the computation of the ΔR's.

COMPUTATION OF THE DERIVATIVES OF Z1

As it is implied by (2), the computation of Z_1 requires the knowledge of the reflection coefficient Γ_1, which is obtained by the recursive procedure presented in [1]. Equations (5) and (6), along with figure 1, illustrate that procedure.

$$\Gamma_N = 0 \tag{5.a}$$

$$\Gamma_n = -\frac{Z_{0n}\;K_0(\beta_n\,r_n) + Z_{n+1}\;K_0'(\beta_n\,r_n)}{Z_{0n}\;I_0(\beta_n\,r_n) + Z_{n+1}\;I_0'(\beta_n\,r_n)} \qquad \text{for } 1 \le n \le (N-1) \tag{5.b}$$

$$Z_n = -\,Z_{0n}\,\frac{K_0(\beta_n\,r_{n-1}) + \Gamma_n\;I_0(\beta_n\,r_{n-1})}{K_0'(\beta_n\,r_{n-1}) + \Gamma_n\;I_0'(\beta_n\,r_{n-1})} \qquad \text{for } 1 \le n \le N \tag{6}$$

where $Z_{0n} = \dfrac{\beta_n}{\sigma_n}$ (7.a)

$$\beta_n = \sqrt[2]{\lambda^2 + j\,\omega\,\mu\,\sigma_n}\,, \tag{7.b}$$

r_n is the outer radius of zone n, Γ_n is the reflection coefficient in zone n, and Z_n is the wave impedance evaluated at the inner radius of zone n (r_{n-1}).

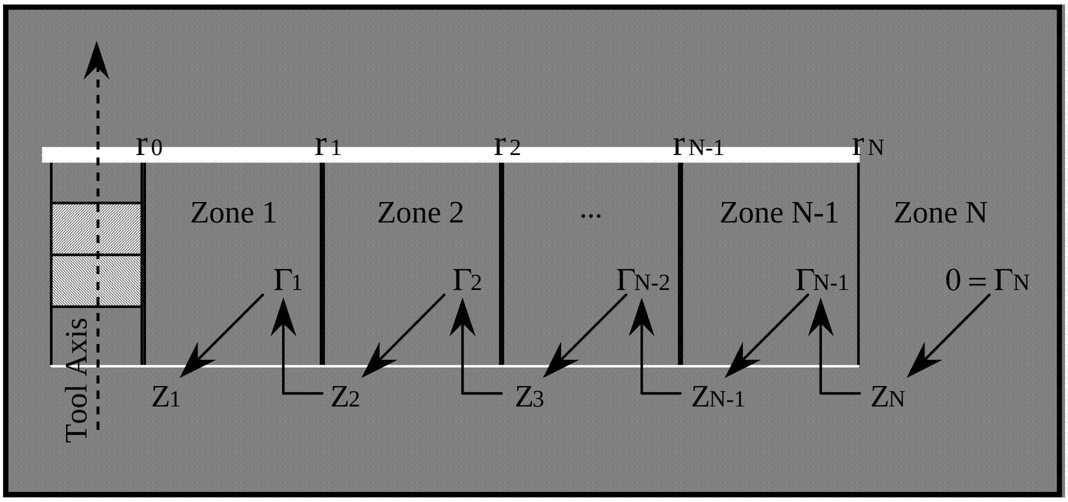

Figure 1: Earthen formation and the computation of Z_1.

Notice from figure 1, that the recursive procedure starts at the outermost zone, where the reflection coefficient is equated to zero (5.a). Then, (5.a) is replaced into (6) in order to compute Z_N (the wave impedance at the outermost boundary r_{N-1}). This value is then used in (5.b) to compute the reflection coefficient in zone N-1, which is again replaced into (6) to obtain Z_{N-1}. In this way, the iterations are continued until zone 1 is reached and the wave impedance at the current element's surface is obtained.

In order to compute the derivatives of Z_1, the recursions described above must be taken into consideration. Then, starting from Z_1 and using the chain rule, differentiation is performed until reaching the zone whose conductivity is being used as the derivative's variable. By doing so, the derivative of Z_1 with respect to the conductivity of zone k will be given by an expression of the form:

$$\frac{\partial Z_1}{\partial \sigma_k} = \frac{\partial Z_1}{\partial \Gamma_1} \frac{\partial \Gamma_1}{\partial Z_2} \frac{\partial Z_2}{\partial \Gamma_2} \cdots \frac{\partial \Gamma_{k-1}}{\partial Z_k} \frac{\partial Z_k}{\partial \sigma_k} \tag{8}$$

which can also be computed by using a recursive procedure.

Before starting the computation of the derivatives, let us remind the following Bessel function identities, which will be required during the computations:

$$K_0'(x) = -K_1(x) \qquad \text{and} \qquad I_0'(x) = I_1(x) \tag{9.a}$$

$$K_1'(x) = -K_0(x) - \frac{K_1(x)}{x} \qquad \text{and} \qquad I_1'(x) = I_0(x) - \frac{I_1(x)}{x} \tag{9.b}$$

where I_1 and K_1 are the first order Modified Bessel functions of first and second kind.

Also, the following derivatives will simplify the computations:

$$\frac{\partial \beta_n}{\partial \sigma_k} = 0 \qquad \forall\ n \neq k \tag{10.a}$$

$$\frac{\partial \beta_k}{\partial \sigma_k} = \frac{j\,\omega\,\mu}{2\,\beta_k} \tag{10.b}$$

$$\frac{\partial K_0(\beta_k\, r_i)}{\partial \sigma_k} = -\frac{j\, r_i\, \omega\, \mu}{2\,\beta_k}\ K_1(\beta_k\, r_i) \tag{11.a}$$

$$\frac{\partial I_0(\beta_k\, r_i)}{\partial \sigma_k} = \frac{j\, r_i\, \omega\, \mu}{2\,\beta_k}\ I_1(\beta_k\, r_i) \tag{11.b}$$

$$\frac{\partial K_1(\beta_k\, r_i)}{\partial \sigma_k} = -\frac{j\, r_i\, \omega\, \mu}{2\,\beta_k}\ K_0(\beta_k\, r_i) - \frac{j\,\omega\,\mu}{2\,\beta_k^2}\ K_1(\beta_k\, r_i) \tag{11.c}$$

$$\frac{\partial I_1(\beta_k\, r_i)}{\partial \sigma_k} = \frac{j\, r_i\, \omega\, \mu}{2\,\beta_k}\ I_o(\beta_k\, r_i) - \frac{j\,\omega\,\mu}{2\,\beta_k^2}\ I_1(\beta_k\, r_i) \tag{11.d}$$

where r_i can be either r_k or r_{k-1}.

Now let us develop the recursive procedure for the evaluation of (8). This will be done by considering the derivatives of the reflection coefficient Γ_n and the wave impedance Z_n, as defined in (5) and (6), with respect to the conductivity of zone k (for $1 \leq k \leq N$). As it will be seen next, depending on the values of n and k, three different kind of derivatives can result:

1.- <u>Null Derivatives</u>. As it can be observed from (5.b) and (6), Z_n and Γ_n are always functions of conductivities n, n+1, ... N. Then, for values of k smaller than n, the derivatives of Z_n and Γ_n with respect to the conductivity k are always zero. This can be easily verified from figure 1.

$$\frac{\partial \Gamma_n}{\partial \sigma_k} = 0 \qquad \text{if } \; n > k \tag{12.a}$$

$$\frac{\partial Z_n}{\partial \sigma_k} = 0 \qquad \text{if } \; n > k \tag{13.a}$$

2.- Long Derivatives. Similarly, it can be seen from (5), (6) and (7), that the dependence on conductivity n appears in all the β_n's present in the expression. Then, for k equal to n, differentiation of (5) and (6) becomes very messy. For practical reasons and because of the embroilment of the algebra, all the intermediate steps are going to be omitted. After differentiating, applying the chain rule, gathering terms and substituting expressions from (5), (6), (9), (10) and (11); the following results are obtained:

$$\frac{\partial \Gamma_k}{\partial \sigma_k} = 0 \qquad \text{if } \; k = N \tag{12.b.1}$$

$$\begin{aligned} \frac{\partial \Gamma_k}{\partial \sigma_k} = & -\frac{j\, r_k\, \mu}{2\, \beta_k} \left\{ \left(\omega\, Z_{k+1} + \frac{\omega}{\sigma_k\, r_k} + \frac{j\, 2\, Z_{0k}^2}{\mu\, r_k} \right) \left[\frac{K_0(\beta_k\, r_k) + \Gamma_k\, I_0(\beta_k\, r_k)}{Z_{0k}\, I_0(\beta_k\, r_k) + Z_{k+1}\, I_0'(\beta_k\, r_k)} \right] \right. \\ & \left. + \, \omega \left(Z_{0k} - \frac{Z_{k+1}}{\beta_k\, r_k} \right) \left[\frac{K_0'(\beta_k\, r_k) + \Gamma_k\, I_0'(\beta_k\, r_k)}{Z_{0k}\, I_0(\beta_k\, r_k) + Z_{k+1}\, I_0'(\beta_k\, r_k)} \right] \right\} \qquad \text{if } \; k < N \end{aligned} \tag{12.b.2}$$

$$\begin{aligned} \frac{\partial Z_k}{\partial \sigma_k} = & \left(\frac{j\, \omega\, \mu}{2\, \beta_k^2} - \frac{1}{\sigma_k} \right) Z_k + \frac{j\, \omega\, \mu\, r_{k-1}}{2\, \beta_k} \left(\frac{Z_k^2}{Z_{0k}} + \frac{Z_k}{\beta_k\, r_{k-1}} - Z_{0k} \right) \\ & - \left[\frac{Z_{0k}\, I_0(\beta_k\, r_{k-1}) + Z_k\, I_0'(\beta_k\, r_{k-1})}{K_0'(\beta_k\, r_{k-1}) + \Gamma_k\, I_0'(\beta_k\, r_{k-1})} \right] \frac{\partial \Gamma_k}{\partial \sigma_k} \end{aligned} \tag{13.b}$$

3.- Short Derivatives. The third kind of derivatives appears for those cases in which k is greater than n. Again, it can be noticed from (5), (6) and (7), that dependence on conductivity k occurs only in the wave impedance or the reflection coefficient functions in the fractional expressions. Then, after applying the chain rule, gathering terms and substituting expressions from (5), (6), (9), (10) and (11); the following results are obtained:

$$\frac{\partial \Gamma_n}{\partial \sigma_k} = -\left[\frac{K_0'(\beta_n r_n) + \Gamma_n I_0'(\beta_n r_n)}{Z_{0n} I_0(\beta_n r_n) + Z_{n+1} I_0'(\beta_n r_n)}\right] \frac{\partial Z_{n+1}}{\partial \sigma_k} \qquad \text{if } \; n < k \tag{12.c}$$

$$\frac{\partial Z_n}{\partial \sigma_k} = -\left[\frac{Z_{0n} I_0(\beta_n r_{n-1}) + Z_n I_0'(\beta_n r_{n-1})}{K_0'(\beta_n r_{n-1}) + \Gamma_n I_0'(\beta_n r_{n-1})}\right] \frac{\partial \Gamma_n}{\partial \sigma_k} \qquad \text{if } \; n < k \tag{13.c}$$

Finally, the recursive implementation of (8) is performed by successive iterations on (5), (6), (12) and (13). Such procedure is clearly illustrated by the flow diagram presented in figure 2.

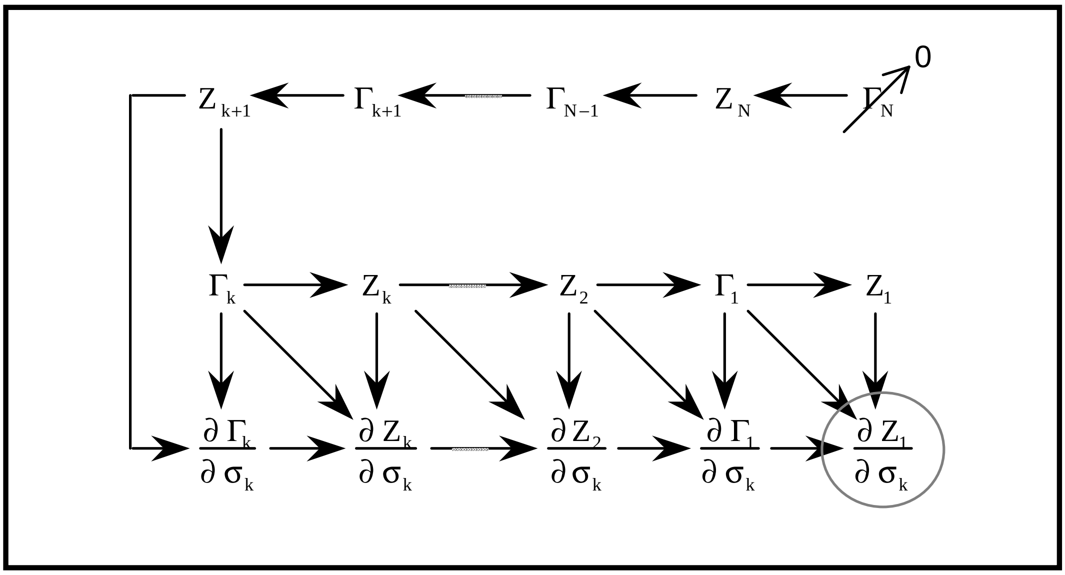

Figure 2: Recursive procedure for computing $\partial Z_1 / \partial \sigma_k$.

NUMERICAL CONSIDERATIONS

Because of the exponential nature of the modified Bessel functions, overflow and underflow conditions can occur when evaluating them at large argument values during the numerical computation of $\partial Z_1 / \partial \sigma_k$. In order to avoid this problem, it is possible to remove the exponential dependencies from the Bessel functions and consider them separately. Then, the functions and their derivatives must be expressed in product form as follows:

$$K_0(\beta_i\, r_j) = \tilde{K}_0(\beta_i\, r_j)\ e^{-\beta_i\, r_j} \tag{14.a}$$

$$K'_0(\beta_i\, r_j) = \tilde{K}'_0(\beta_i\, r_j)\ e^{-\beta_i\, r_j} \tag{14.b}$$

$$I_0(\beta_i\, r_j) = \tilde{I}_0(\beta_i\, r_j)\ e^{+\beta_i\, r_j} \tag{14.c}$$

$$I'_0(\beta_i\, r_j) = \tilde{I}'_0(\beta_i\, r_j)\ e^{+\beta_i\, r_j} \tag{14.d}$$

where all the tilded functions do not have exponential dependencies.

Then, let us rewrite the recursive equations presented in the previous section in terms of the tilded functions in (14). Let us start by replacing (14) into (5). By doing so, the following expression is obtained:

$$\Gamma_n = \left[- \frac{Z_{0\,n}\ \tilde{K}_0(\beta_n\, r_n) + Z_{n+1}\ \tilde{K}'_0(\beta_n\, r_n)}{Z_{0n}\ \tilde{I}_0(\beta_n\, r_n) + Z_{n+1}\ \tilde{I}'_0(\beta_n\, r_n)} \right] \frac{e^{-\beta_n\, r_n}}{e^{+\beta_n\, r_n}} \qquad \text{for } 1 \le n \le (N-1) \tag{15}$$

where the term in brackets is going to be denoted as $\tilde{\Gamma}_n$, so that:

$$\Gamma_n = \tilde{\Gamma}_n\ e^{-2\beta_n\, r_n} \qquad \text{for } 1 \le n \le (N-1) \tag{16}$$

In this way (5) is then replaced by:

$$\tilde{\Gamma}_N = 0 \tag{17.a}$$

$$\tilde{\Gamma}_n = - \frac{Z_{0\,n}\ \tilde{K}_0(\beta_n\, r_n) + Z_{n+1}\ \tilde{K}'_0(\beta_n\, r_n)}{Z_{0n}\ \tilde{I}_0(\beta_n\, r_n) + Z_{n+1}\ \tilde{I}'_0(\beta_n\, r_n)} \qquad \text{for } 1 \le n \le (N-1) \tag{17.b}$$

Next, by substituting (14) and (16) into (6), the new version of (6) is obtained:

$$Z_n = - Z_{0n}\ \frac{\tilde{K}_0(\beta_n\, r_{n-1}) + \tilde{\Gamma}_n\ e^{-2\,\beta_n\,(r_n - r_{n-1})}\ \tilde{I}_0(\beta_n\, r_{n-1})}{\tilde{K}'_0(\beta_n\, r_{n-1}) + \tilde{\Gamma}_n\ e^{-2\,\beta_n\,(r_n - r_{n-1})}\ \tilde{I}'_0(\beta_n\, r_{n-1})} \qquad \text{for } 1 \le n \le N \tag{18}$$

Similarly, by using (14) and (16) into (12) and (13), new expressions for them are gotten:

1.- <u>Null Derivatives</u>. ($k < n$)

$$\frac{\partial\,\tilde{\Gamma}_n}{\partial\,\sigma_k} = 0 \tag{19.a}$$

$$\frac{\partial\,Z_n}{\partial\,\sigma_k} = 0 \tag{20.a}$$

2.- Long Derivatives. (k = n)

$$\frac{\partial\,\tilde{\Gamma}_k}{\partial\,\sigma_k} = 0 \qquad \text{if} \quad k = N \tag{19.b.1}$$

$$\frac{\partial\,\tilde{\Gamma}_k}{\partial\,\sigma_k} = -\frac{j\,r_k\,\mu}{2\,\beta_k}\left\{\left(\omega\,Z_{k+1} + \frac{\omega}{\sigma_k\,r_k} + \frac{j\,2\,Z_{0k}^2}{\mu\,r_k}\right)\left[\frac{\tilde{K}_0(\beta_k\,r_k) + \tilde{\Gamma}_k\,\tilde{I}_0(\beta_k\,r_k)}{Z_{0k}\,\tilde{I}_0(\beta_k\,r_k) + Z_{k+1}\,\tilde{I}_0'(\beta_k\,r_k)}\right]\right.$$
$$\left. + \,\omega\left(Z_{0k} - \frac{Z_{k+1}}{\beta_k\,r_k}\right)\left[\frac{\tilde{K}_0'(\beta_k\,r_k) + \tilde{\Gamma}_k\,\tilde{I}_0'(\beta_k\,r_k)}{Z_{0k}\,\tilde{I}_0(\beta_k\,r_k) + Z_{k+1}\,\tilde{I}_0'(\beta_k\,r_k)}\right]\right\} \qquad \text{if} \quad k < N \tag{19.b.2}$$

$$\frac{\partial\,Z_k}{\partial\,\sigma_k} = \left(\frac{j\,\omega\,\mu}{2\,\beta_k^2} - \frac{1}{\sigma_k}\right) Z_k + \frac{j\,\omega\,\mu\,r_{k-1}}{2\,\beta_k}\left(\frac{Z_k^2}{Z_{0k}} + \frac{Z_k}{\beta_k\,r_{k-1}} - Z_{0k}\right)$$
$$- \left[\frac{Z_{0\,k}\,\tilde{I}_0(\beta_k\,r_{k-1}) + Z_k\,\tilde{I}_0'(\beta_k\,r_{k-1})}{\tilde{K}_0'(\beta_k\,r_{k-1}) + \tilde{\Gamma}_k\,e^{-2\,\beta_k\,(r_k - r_{k-1})}\,\tilde{I}_0'(\beta_k\,r_{k-1})}\right] e^{-2\,\beta_k\,(r_k - r_{k-1})}\,\frac{\partial\,\tilde{\Gamma}_k}{\partial\,\sigma_k} \tag{20.b}$$

3.- Short Derivatives. (k > n)

$$\frac{\partial\,\tilde{\Gamma}_n}{\partial\,\sigma_k} = -\left[\frac{\tilde{K}_0'(\beta_n\,r_n) + \tilde{\Gamma}_n\,\tilde{I}_0'(\beta_n\,r_n)}{Z_{0\,n}\,\tilde{I}_0(\beta_n\,r_n) + Z_{n+1}\,\tilde{I}_0'(\beta_n\,r_n)}\right]\frac{\partial\,Z_{n+1}}{\partial\,\sigma_k} \tag{19.c}$$

$$\frac{\partial\,Z_n}{\partial\,\sigma_k} = -\left[\frac{Z_{0n}\,\tilde{I}_0(\beta_n\,r_{n-1}) + Z_n\,\tilde{I}_0'(\beta_n\,r_{n-1})}{\tilde{K}_0'(\beta_n\,r_{n-1}) + \tilde{\Gamma}_n\,e^{-2\,\beta_n\,(r_n - r_{n-1})}\,\tilde{I}_0'(\beta_n\,r_{n-1})}\right] e^{-2\,\beta_n\,(r_n - r_{n-1})}\,\frac{\partial\,\tilde{\Gamma}_n}{\partial\,\sigma_k} \tag{20.c}$$

In this way, the recursive implementation of (8) can be performed in the same manner illustrated in figure 2, but using equations (17), (18), (19) and (20) instead of (5), (6), (12) and (13). In this new procedure, numerical overflow and underflow during the evaluation of the Bessel functions are less likely to occur.

PROGRAMMING STRATEGIES

Due to the complexity of the equations involved in the computation of $\partial Z_1 / \partial \sigma_k$ it would be very helpful to define some common factors and rewrite the recursive equations in terms of them. By looking carefully at (17), (18), (19) and (20), the following common functions or subroutines can be defined:

$$g_{kk}(a,b) = Z_{0a}\tilde{K}_0(\beta_a r_b) + Z_{b+1}\tilde{K}_0'(\beta_a r_b) \quad (21.a)$$

$$g_{ii}(a,b) = Z_{0a}\tilde{I}_0(\beta_a r_b) + Z_{b+1}\tilde{I}_0'(\beta_a r_b) \quad (21.b)$$

$$f_{ki}(a,b,\chi) = \tilde{K}_0(\beta_a r_b) + \tilde{\Gamma}_a \chi \tilde{I}_0(\beta_a r_b) \quad (21.c)$$

$$d_{ki}(a,b,\chi) = \tilde{K}_0'(\beta_a r_b) + \tilde{\Gamma}_a \chi \tilde{I}_0'(\beta_a r_b) \quad (21.d)$$

where a and b are the input variables and χ is an output variable given by:

$$\chi = \chi(a,b) = e^{-2\beta_a (r_a - r_b)} \quad (21.e)$$

In this way, (17), (18), (19) and (20) can be expressed in terms of the functions in (21) as follows:

$$\tilde{\Gamma}_n = \begin{cases} 0 & \text{if } n = N \\ -\dfrac{g_{kk}(n,n)}{g_{ii}(n,n)} & \text{otherwise} \end{cases} \quad (22)$$

$$Z_n = -Z_{0n}\frac{f_{ki}(n,n-1,\chi)}{d_{ki}(n,n-1,\chi)} \quad (23)$$

$$
\frac{\partial \tilde{\Gamma}_n}{\partial \sigma_k} = \begin{cases} 0 & \text{if } n > k \text{ or } n = k = N \\ -\dfrac{j\,\mu\, r_k}{2\,\beta_k}\left\{\left(\omega\, Z_{k+1} + \dfrac{\omega}{\sigma_k\, r_k} + \dfrac{j\,2\,Z_{0k}^2}{\mu\, r_k}\right)\dfrac{f_{ki}\,(k,k,\chi)}{g_{ii}\,(k,k)} \right. & \\ \qquad \left. + \,\omega\left(Z_{0k} - \dfrac{Z_{k+1}}{\beta_k\, r_k}\right)\dfrac{d_{ki}\,(k,k,\chi)}{g_{ii}\,(k,k)}\right\} & \text{if } n = k \neq N \\ -\dfrac{d_{ki}\,(n,n,\chi)}{g_{ii}\,(n,n)}\ \dfrac{\partial Z_{n+1}}{\partial \sigma_k} & \text{if } n < k \end{cases} \tag{24}
$$

$$
\frac{\partial Z_n}{\partial \sigma_k} = \begin{cases} 0 & \text{if } n > k \\ \left(\dfrac{j\,\omega\,\mu}{2\,\beta_k^2} - \dfrac{1}{\sigma_k}\right) Z_k + \dfrac{j\,\omega\,\mu\, r_{k-1}}{2\,\beta_k}\left(\dfrac{Z_k^2}{Z_{0k}} + \dfrac{Z_k}{\beta_k\, r_{k-1}} - Z_{0k}\right) & \\ \qquad - \dfrac{\chi\ g_{ii}\,(k,k-1)}{d_{ki}\,(k,k-1,\chi)}\ \dfrac{\partial \tilde{\Gamma}_k}{\partial \sigma_k} & \text{if } n = k \\ -\dfrac{\chi\ g_{ii}\,(n,n-1)}{d_{ki}\,(n,n-1,\chi)}\ \dfrac{\partial \tilde{\Gamma}_n}{\partial \sigma_k} & \text{if } n < k \end{cases} \tag{25}
$$

Finally, a pseudo code for the computation of $\partial Z_1 / \partial \sigma_k$ is presented.

$$
\begin{array}{l}
\text{subroutine} \quad dZ_1 / d\sigma_k \,(k, N) \quad \{ \\
\text{double complex } \tilde{\Gamma}(1:N), Z(1:N), d\tilde{\Gamma}(1:N), dZ(1:N) \\
\text{for} \quad n = N \text{ to } 1 \qquad \text{step size } -1 \\
\qquad \tilde{\Gamma}(n) = \tilde{\Gamma}_n \\
\qquad Z(n) = Z_n \\
\qquad \text{if} \quad (n \le k) \quad \text{then} \\
\qquad\qquad d\tilde{\Gamma}(n) = \dfrac{\partial \tilde{\Gamma}_n}{\partial \sigma_k} \\
\qquad\qquad dZ(n) = \dfrac{\partial Z_n}{\partial \sigma_k} \\
\qquad \text{end if} \\
\text{end for} \\
\text{return} \quad dZ(1) \quad \}
\end{array}
$$

Algorithm 1: Recursive computation of $\partial Z_1 / \partial \sigma_k$.

Two important considerations must be taken into account when using the algorithm shown above. The first one is that the computations of $\tilde{\Gamma}_n$, Z_n, $\partial \tilde{\Gamma}_n / \partial \sigma_k$ and $\partial Z_n / \partial \sigma_k$ must be done as defined in (22), (23), (24) and (25) respectively. And the second is that according to the algorithm, the computation of Z_1 is obtained by setting the input variable k to 0.

CONCLUSIONS

As it was mentioned before, the availability of an analytic procedure for the computation of the tool measurement’s first order derivatives improves the convergence and the efficiency (in terms of computational time) of the Time Harmonic Field Electric Logging inverse problem. The use of a linear inversion technique only requires the knowledge of the first order derivatives, and because of the smoothness of the measurement function, linear approximations should perform a very good work most of the time. However, in case that a different kind of inversion technique (that uses higher order derivatives) would be required, we will certainly get a better numerical approximation of the higher order derivatives if starting from the analytically-computed first derivatives than if starting from the measurement function itself.

REFERENCES

[1] Bostick, F.; Smith, H. (1994), Propagation Effects in Electric Logging. University of Texas at Austin.

[2] Update Report #12: The Inverse Problem. (In Progress)

[3] Update Report #5: Solution of the Potential Difference Integral by Using Exponential Windowing.