The Inverse Problem

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INTRODUCTION

This report presents a general discussion about the strategies and methodologies that will be used in the inverse version of the time harmonic field electric logging problem (THFEL). In the inverse version we are interested in obtaining the earthen formation parameters given a set of measurements and the tool configuration. Due to the very complex non-linear relationships between the formation parameters and the logging tool measurements, it is practically impossible to compute or approximate an acceptable solution by using direct inversion techniques. For this reason, we will concentrate our attention in inverse modeling.

INVERSE MODELING

Inverse modeling, also known as iterative inversion, is a procedure in which the forward or direct problem is repeatedly solved for a given model, which is updated at each iteration, until some optimization criterion is achieved. The success of an inverse modeling procedure depends on the assertive choice and appropriate interaction of six basic elements of the inversion process [1]. Although the specific characteristics of most of those elements are generally determined by the properties of the particular problem under consideration, there is always a plenty of alternatives that can be considered in order to improve the performance of the inverse modeling procedure.

Next, those six basic elements of the inversion process are described. Figure 1 illustrates the relationships among them.

1.- <u>Inversion Data.</u>

The inversion data consists of a set of quantitative observations or measurements that are used as the input data for the inversion process. The inversion data can be experimental (recollected from the real physical problem) or theoretical (generated analytically by a simulation of the physical problem). In practice, experimental data is generally contaminated with noise. The quality of an inversion data set is determined basically by the amount of non-redundant information contained in it, which is of great importance for the success of the inversion process.

2.- A Priori Information.

Consists of any additional knowledge about particular properties or conditions of the problem. Generally, a priori information can help to chose the most appropriate definitions for other of the elements in the inversion process, as for example the inversion model and the objective function. A priori information can be of qualitative nature as well as of quantitative nature.

3.- Inversion Model.

The inversion model consists of the set of unknowns which are to be estimated by the inversion procedure. In other words, it represents the problem's physical properties that we are interested in compute. Such a set is often referred as the model parameters or, simply, the model. The space defined by all possible combinations of model parameters is called the model space. In many practical situations, the physical properties represented by the model are continuos functions of space, time, or any other variable; and some parametrizations are required. Parametrization can be accomplished in many different ways by using techniques such as splines, wavelets, Fourier transforms, discrete representations, etc... The appropriate choice of an inversion model is very important for the performance of the inversion procedure. A good model must be as simple as possible while providing a reliable representation of the physical problem's parameters.

4.- Forward Modeling Algorithm.

The forward modeling algorithm is a procedure for computing the response of a given model. It generates synthetic data by implementing a theoretical simulation of the physical problem [2]. The forward modeling algorithm can be interpreted as a multidimensional function that maps the model space into a different space that is called the data, or solution, space. Notice that the inversion data set is one point of the solution space. The availability of a good forward modeling algorithm is of great importance for the success of the inversion process. Deficient algorithms generally lead to modelization errors that deteriorate the performance of the inversion procedure.

5.- Objective Function.

The objective function, also called error, cost or fitness function, provides a measurement of misfit between a given model and a possible solution model. It compares the response of the given model (synthetic data) with the inversion data set. Notice that the objective function does not necessarily identifies the correct solution model. This is because, as it will be seen later, the correspondence between points in the model and data spaces is not necessarily unique. However, objective functions are very useful for identifying good solution models. Generally, they are defined in such a way that good solutions are located at the minima. So, the optimization problem is transformed into the minimization problem of the objective function. A good selection of the objective function is very important for the success of the inversion process because it assists to reduce the amount of spurious solutions and to emphasize the differences between them and the actual one.

6.- Inverse Modeling Algorithm.

The inverse modeling algorithm, also called the method of search, consists of a set of rules which objective is, starting form an initial model, to find a better one into the model space. By a better model, we refer to a model which response is closer to the inversion data set (according to the objective function) than the response of the initial one. There exist a huge variety of methods of search; but they can be cataloged into two main categories, which are the global methods and the local methods. They will be discussed in more detail later. Again the choice of the inverse modeling algorithm plays an important role in the success of the inversion process. Generally, the best choice is determined by the nature of the problem itself an factors as the quality of the inversion data set, existent a priori information, computational complexity of the forward modeling algorithm, availability of the derivatives, etc...

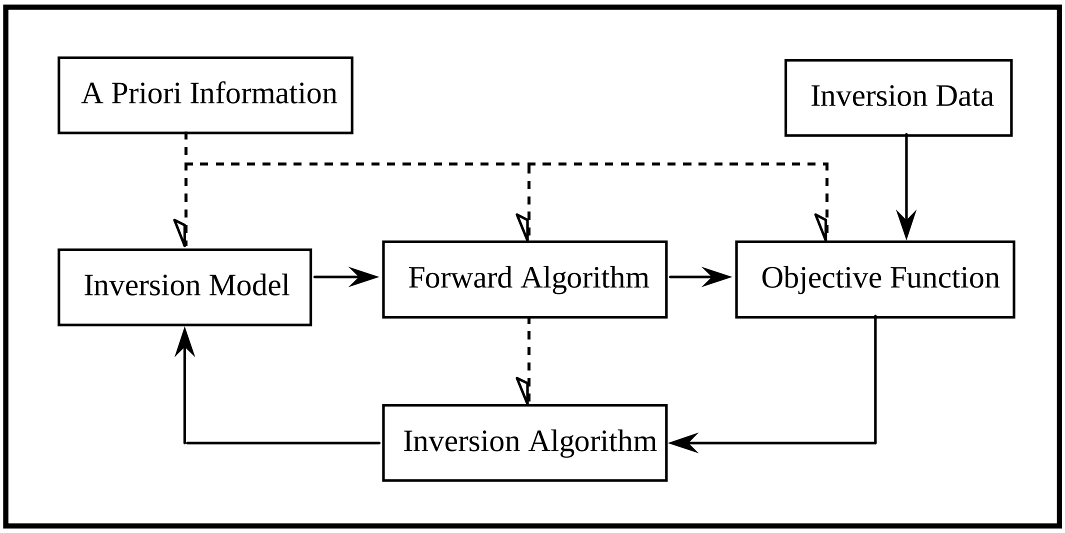

Figure 1: Elements of an Inverse Modeling Procedure.

In summary, as it can be seen at Figure 1, the inverse modeling process consists of a recursive procedure in which the inversion model is updated at each iteration. The final goal of the inversion process is to find an image in the model space (an inversion model) for the given inversion data; in other words, a point in the model space such that the forward modeling algorithm would map it into the solution space as the given inversion data.

THE EARTHEN FORMATION MODEL

The general earthen formation model to be used for the inversion of the THFEL problem is presented in Figure 2. The discretization of the earthen formation into concentric cylindrical zones can be justified by the fact that clearly differentiable types of zones actually occur in the practice. They are the borehole, the mud cake, the invaded formation and the real formation. Although each of them are not homogeneous zones, their properties can be considered to present less variations inside themselves than among them. However, the number of zones in the model may be always increased in order to provide a better representation of the actual formation.

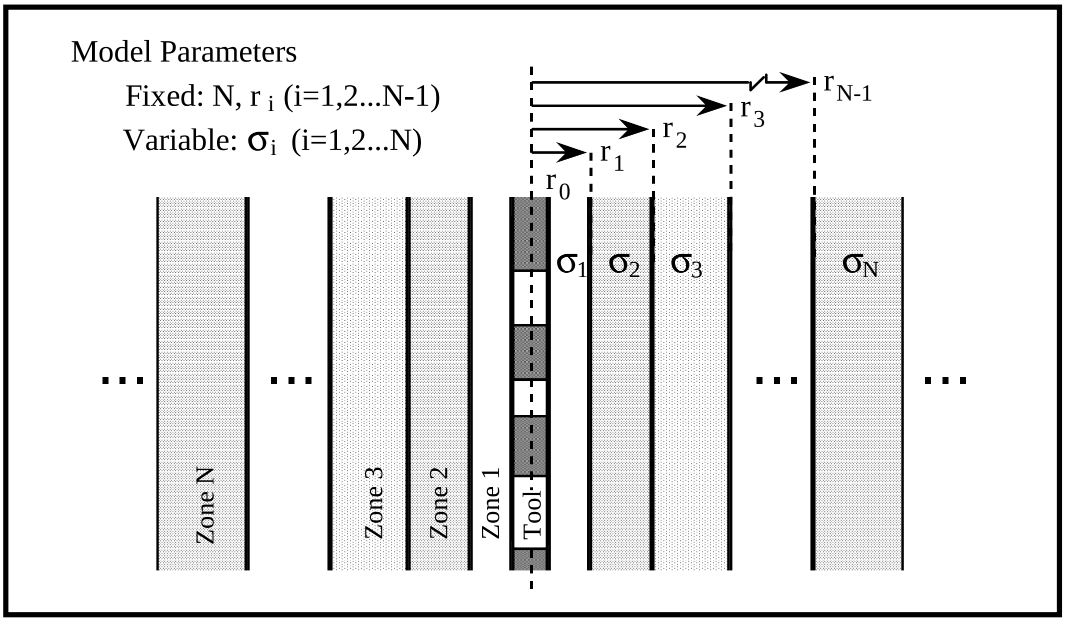

Figure 2: Earthen Formation Model.

Notice from Figure 2 that the zone's conductivities are the only model parameters considered as unknowns. The radii of the boundaries, on the other hand, are considered to be known and will not constitute inversion model parameters. So, they have to be defined with a reasonable value. The earthen formation model has been defined in this way in order to avoid the non-linearities of great complexity introduced by radii variations. Notice, however, that the limitations introduced by fixing the values of the radii may be always overcome by increasing the number of zones in the model.

Additional a priori information can be also used to improve the inversion model. For example, the knowledge of the borehole radius can be incorporated into the model; and the knowledge of the mud conductivity may be used to define a better starting model. In this way, a more accurate representation of the earthen formation will be provided and a better performance from the inversion procedure will be obtained.

LOCAL SEARCH ALGORITHMS

Local search algorithms start searching from a pre-defined initial model and use the information in the local derivatives of the objective function to update the model at each iteration. The objective function or error surface is $R^n \Rightarrow R$, where n is the dimensionality of the data space. The goal of a local search algorithm is to find the global minimum of the error surface; however that may not be necessarily accomplished because there will be always a risk for the algorithm to get trapped into a local minimum.

Gradient methods [3], iterative Born approximation [4] and Newton methods are among the most popular local search algorithms. Only the first two are going to be implemented for the THFEL inversion procedure. That is because they only rely on the first derivatives of the objective function, which can be analytically approximated [5]. On the other hand, other local search algorithms that require second order derivatives, such as the Newton methods, are not a good option for the THFEL inversion procedure because of their extremely expensive cost from a computational point of view.

In general, local search algorithms consists of a loop of iterations that is terminated when certain stopping criterion is achieved. At each iteration, three basic steps are performed. First, the objective function and its derivatives are evaluated for the current model. Second, a jump in the model space is computed by using the information provided by the objective function and its derivatives. And third, a new model obtained by adding the jump to the previous one. The stopping criterion consists of a set of conditions that determines when the updated model could represent a valid solution.

GLOBAL SEARCH ALGORITHMS

Global search algorithms perform their search by moving through the model space following a set of rules with certain random foundation. They can perform a purely random search as it is the case of Random Walks, or do it with some directivity as it is the case of Simulated Annealing [6] and Genetic Algorithms [7]. In general, they do not require knowledge of the objective function derivatives. The only information they need is the value of the objective function itself.

According to the results presented in [1], simulated annealing and genetic algorithms have proven to be good alternatives in geophysical inversion problems. For this reason, they are the global search algorithms that will be implemented for the THFEL inversion procedure.

Simulated annealing algorithms are based on the analogy between the problem of finding the minimum of a function of multiple variables and the statistical mechanics phenomenon of annealing [1]. In this kind of optimization technique, the randomness of the search is controlled by a parameter called the acceptance temperature. In this way, at the beginning of the execution, the algorithm searches randomly all over the objective function. Then, as the acceptance temperature is decreased, the searching process tends to get concentrated in certain region; but always with the eventual chance of jumping away. One of the most important features of simulated annealing is that there is always a possibility of escaping from a local minimum.

Genetic algorithms, on the other hand, are based on the analogy between the way biological communities evolve and the problem of maximizing a function of multiple variables. They perform their search by considering a ‘population’ of models instead of a single model at a time. The best fitted models, according to the objective function, are selected at random to be combined and create a new generation of models. In the same way that the process of natural selection improves the average performance of a biological population after some generations, genetic algorithms will improve the average fitting of a set of models after certain amount of iterations.

Global optimization methods are in general more robust than local methods. In fact, they are less vulnerable to get stuck into local minima because they perform a more exhaustive search than local search techniques do. However, they present the disadvantage of being much more intensive from the computational point of view.

OTHER CONSIDERATIONS ABOUT THE THFEL PROBLEM

The time harmonic field electric logging problem, which is described in [2], has some important properties that must be considered in order to define the most appropriate alternatives for the inversion procedure.

The most important peculiarities of this problem are its complexity and non-linearity. The measurements registered at the logging tool are the result of the combination of the primary field injected by the tool and all its reflections and multiple reflections coming back from the earthen formation. So the resulting measurement is a very complicated non-linear function of the formation parameters, the tool configuration and its frequency of operation.

Under those premises, global optimization methods seem to be the most appropriate alternative as inverse modeling algorithms; however, the big computational cost involved in these kind of algorithms makes them impractical in most of the cases. On the other hand, local search methods, if provided with a good starting point, prove to be very efficient and reliable algorithms for the solution of the THFEL inverse problem. The problem is that a good starting model cannot be always provided. Nevertheless, it is possible to exploit the benefits of both global and local methods by using an hybrid optimization scheme. This last option seems to be the most suitable alternative [1].

Another property of the THFEL problem is the non unique correspondence between the formation model parameters and the tool measurements. This means that it is possible for the same set of measurements to represent the response of different formation models. However, this problem can always be reduced by increasing the number of linearly independent measurements in the inversion data set.

As a final remark, it is important to mention the fact that due to the unavailability of experimental data, the inversion data to be used for the inversion will be generated analytically by the forward modeling algorithm. Uniformly and normal distributed noise will be eventually added to the theoretical data in order to make the simulations more realistic and to provide means for evaluating the performance of the inversion algorithms.

CONCLUSIONS

The complexity and non-linearity of the THFEL problem make inverse modeling to be the most appropriate option. Proper choice of each of the elements involved plays a very important role in the success of the inversion procedure.

The inversion model has been defined by a set of conductivities corresponding to the discrete representation of an earthen formation shown in Figure 2. As it was explained before, the number of zones and radii are assume to be known and constitute fixed parameters of the model.

Among the existing inverse modeling algorithms, four of them have been considered as possible alternatives. Two global methods, which are simulated annealing and genetic algorithms; and two local methods, which are the Born approximation and gradient methods. Also, the combination of them in hybrid optimization schemes has been considered.

REFERENCES

[1] Chunduru, R. (1996), Global and Hybrid Optimization in Geophysical Inversion. University of Texas at Austin.

[2] Update Report #1: The Direct Problem.

[3] Update Report #13: Gradient Methods.

[4] Update Report #14: Born Approximation.

[5] Update Report #8: Recursive computation of the Derivatives.

[6] Update Report #15: Simulated Annealing.

[7] Update Report #16: Genetic Algorithms.