Modeling of Laterolog Systems

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INTRODUCTION

This report describes how to solve the direct version of the time harmonic field electric logging problem for a laterolog system. The technique is basically the same used for the simple 2-electrode logging tool [1]. Because the same kind of basic current elements are used, the electromagnetic part of the problem is exactly the same [2], but the linear algebra part is different due to the more complex structure of laterolog systems. For this reason, most of the attention will be directed to the linear algebra problem. A detailed analysis of the conditions and properties of laterologs that lead to the required set of equations will be presented. For practical purposes, the particular case of a 9-electrode laterolog system will be considered.

THE 9-ELECTRODE LATEROLOG

A very important kind of logging tools are the so called focused tools. The main purpose of this type of tools is to obtain a more accurate measurement of the formation's resistivity by avoiding the effects of the borehole and the surrounding formations. Focused tools can be separated into two groups: divergent logs, that achieve focusing by using an arrangement of measuring electrodes; and laterologs, that achieve it by using an arrangement of current electrodes [3],[4].

For simplicity, only the 9-electrode laterolog in its shallow configuration is going to be presented. Figure 1 presents an schematic view of a 9-electrode shallow laterolog system (LLs); the left side of it shows the internal circuit of the system, and the right side shows the current distribution in an homogeneous medium. As it can be seen from figure 1, the LLs is composed by four potential monitoring electrodes, M1, N1, M2 and N2; two large current return electrodes, B1 and B2; and three current injection electrodes, A1 and A2 that are called the bucking electrodes and inject the bucking current Ia into the formation, and A0 that is called the survey electrode and injects the survey current Io. As it can also be seen from figure 1, some of these electrodes are short circuited in pairs; B1 is short circuited to B2, A1 to A2, M1 to M2 and N1 to N2.

The operation of the LLs is very simple. It achieves focusing by varying the auxiliary or bucking current Ia until the potential difference between electrodes M1 and N1 (or equivalently M2 and N2) is zero. When this potential is zero the tool is said to be in its normal focused condition, and the survey current Io is forced inside the formation as it is shown in the right side of figure 1.

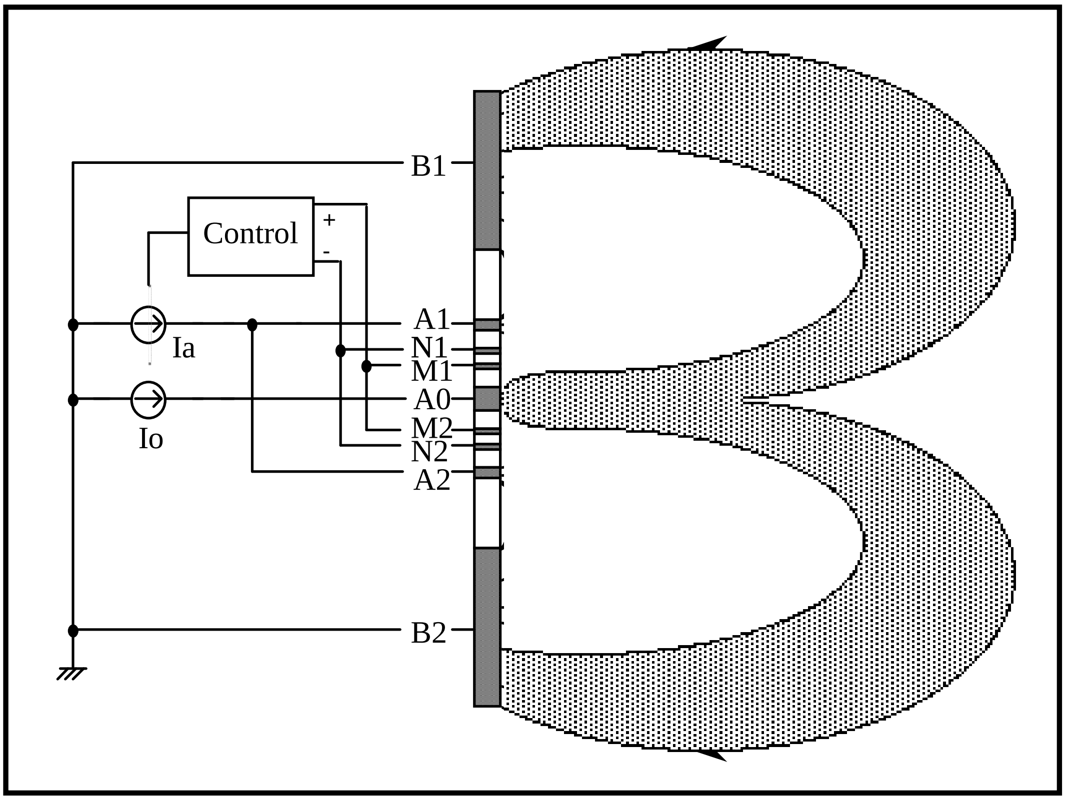

Figure 1: 9-electrode Shallow Laterolog (LLs): Circuit and Current Distribution.

When the formation and/or the borehole parameters change the survey current beam deviates from the normal focused condition. It can be by diverging or converging, depending upon the new borehole-formation characteristics. In the case that the beam diverges, a positive potential would be measured between the monitoring electrodes (M1 and N1) and the controller would respond by increasing the bucking current Ia until the system reaches the normal focused condition again. On the other hand, in the case that the beam converges, a negative potential would be measured between the monitoring electrodes and the controller would decrease the bucking current until reaching the normal focused condition.

Because all four potential monitoring electrodes are at the same potential while operating under normal focused condition, any of them can be used for recording the tool measurement. Then, for example, the tool measurement could be the potential difference between M1 and B1.

It is important to notice that in the solution of the time harmonic field electric logging problem for a laterolog system by using the method of moments [1], in addition to the current strengths of the current elements, the bucking current Ia is also an unknown variable that must be solved for. In the following, we are going to present the different kind of equations that must be considered in the modeling of a laterolog system. Although those kind of equations are valid for any laterolog system, we are going to concentrate specifically on the LLs.

THE LINEAR ALGEBRA PROBLEM IN LATEROLOG SYSTEMS

Once the electromagnetic problem is solved for the basic current element, the overall response of the logging system can be computed by linearly combining a large amount of current elements. This procedure is called the method of moments and was already used in the modeling of the simple (2-electrode) logging tool [1].

In the following sections, the equations that describe the behavior of a laterolog system are described. For simplicity, the representative 9-electrode shallow laterolog system presented in figure 2 is going to be considered. It is important to mention that this tool constitutes just an explanatory model and it does not represent an actual tool. Otherwise, the tool shown in figure 1 offers a better representation of relative sizes and spacing of the electrodes in an actual LLs system.

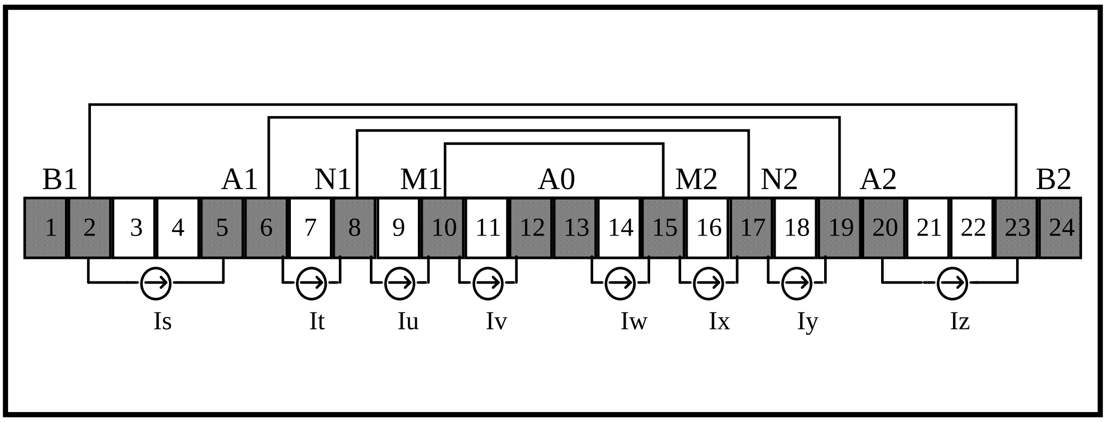

Figure 2: LLs Model to be Considered for Illustrating the Linear Algebra Problem.

As it can be seen from figure 2, the size of the tool to be considered is 24 segment lengths. So, 23 current elements are required for modeling it (remember that the size of each current element is two segment lengths). Those current elements are going to be enumerated as 1 to 23 from left to right and their unknown current strengths as I_1 to I_{23} respectively. Notice also from figure 2 that a total of eight current sources of unknown intensities interconnect the consecutive electrodes. Also, as it was mentioned before, the bucking current Ia is unknown; then, the total number of unknowns for the tool in figure 2 add to 32. Therefore, a total of 32 independent equations must be found.

In solving the linear algebra problem for a laterolog system, five different kinds of equations must be written. Two of them are the same kind of equations used in the case of the simple tool [1] and are going to be referred as the Method of Moment equations. The other three kind of equations are related to the specific characteristics of the laterolog tool configuration and are going to be referred as the System equations.

THE METHOD OF MOMENT EQUATIONS

Let us start with the Method of Moment equations. Two kind of Method of Moment equations are to be considered, they are the electrode equations and the insulator equations. In the electrode equations, the potential difference between centers of consecutive segments over the electrode surfaces are equated to zero [1]. That is because the electrodes are supposed to be made of a perfect conductor. The number of equations in each electrode will depend on the size of the electrode. For an electrode of n segment lengths, there will be n-1 independent equations. Notice from figure 2 that the potential monitoring electrodes are only one segment in length, while all others are two segments in length. Then, no equation can be written for electrodes N1, M1, N2 and M2; and only one electrode equation can be written for electrodes B1, B2, A0, A1 and A2. For example, the equation related to electrode B1 would be:

$$0 = I_1 \Delta R_0 + I_2 \Delta R_1 + I_3 \Delta R_2 + ... + I_{23} \Delta R_{22} \qquad (1)$$

where the I's are the current strengths of the current elements and the ΔR's are the electromagnetic-parameter-dependent resistive quantities defined by equation (49) in [2].

Equation (1) simply states that the potential difference between the centers of segments 1 and 2 is equal to zero; and each term in (1) refers to the contribution of a particular current element to that potential. So, $I_n \Delta R_k$ refers to the contribution of nth current element. Notice that the index of each ΔR is defined by the distance, in segment lengths, between the contributing current element and the segment in which the potential is being measured. Then, in the particular case of (1), all the terms $I_n \Delta R_k$ satisfy k=n-1.

For practical reasons, let us introduce a more compact notation in order to simplify the way of writing the equations. Let us define the parameter ΔP_n as the potential difference between the center of segments n and n+1. More formally, ΔP_n can be defined as follows:

$$\Delta P_n = \sum_{i=1}^{23} \Delta R_{i-n} I_i \qquad (2)$$

where it is important to remember the even symmetry of the ΔR's, that is $\Delta R_{-n} = \Delta R_n$ [2].

Notice that by using this definition, (1) can be expressed just as:

$$\Delta P_1 = 0 \tag{3}$$

In this way, the five electrode equations are given by:

B1: $\Delta P_1 = 0$ (4.a)

B2: $\Delta P_{23} = 0$ (4.b)

A0: $\Delta P_{12} = 0$ (4.c)

A1: $\Delta P_5 = 0$ (4.d)

A2: $\Delta P_{19} = 0$ (4.e)

In the insulator equations, the current strengths of the current elements in between two electrodes are equated to the current flowing trough the tool axis [1]. That is because the tool surface in between electrodes is supposed to be made of a perfect insulator, so there must not be radial current flow. As it can be seen from figure 2, there are eight insulated zones in the tool under consideration. Let us refer to each of those zones by using the pair of electrodes they are in between of. Then, for example, the left most insulated zone is going to be referred as B1A1. The number of equations in each insulated region will depend on the size of the region. For an insulated region of n segment lengths, there will be n+1 independent equations. According to this, as it can be seen from figure 2, three insulator equations can be written for the insulated regions B1A1 and A2B2, and two can be written for each of the other six insulated regions. They are given by:

B1A1: $I_2 = I_3 = I_4 = I_s$ (5.a)

A1N1: $I_6 = I_7 = I_t$ (5.b)

N1M1: $I_8 = I_9 = I_u$ (5.c)

M1A0: $I_{10} = I_{11} = I_v$ (5.d)

A0M2: $I_{13} = I_{14} = I_w$ (5.e)

M2N2: $I_{15} = I_{16} = I_x$ (5.f)

N2A2: $I_{17} = I_{18} = I_y$ (5.g)

A2B2: $I_{20} = I_{21} = I_{22} = I_z$ (5.h)

At this point we have only 23 equations out of the required 32.

THE SYSTEM EQUATIONS

Let us now write the System equations to complete the set of equations. As it was mentioned before, the System equations are defined by the specific laterolog tool configuration and operating conditions. Three different kinds of System equations are to be considered, they are the short-circuit potential equations, the short-circuit current equations and the control condition equations.

In the short-circuit potential equations the fact that short-circuited electrodes are forced to be at the same electric potential is exploited. Let us write the first of such equations by considering the short-circuited electrodes M1 and M2 (see figure 2). Due to the presence of the short circuit, electrodes M1 and M2 must be at the same potential; that means that the potential difference between segments 10 and 15 must be zero. Using the notation introduced in (2), this can be expressed as:

$$\Delta P_{10} + \Delta P_{11} + \Delta P_{12} + \Delta P_{13} + \Delta P_{14} = \sum_{i=10}^{14} \Delta P_i = 0 \quad (6.a)$$

In the same way, equations for the other short-circuited electrodes can be written as follows:

N1N2: $\sum_{i=8}^{16} \Delta P_i = 0$ (6.b)

A1A2: $\sum_{i=6}^{18} \Delta P_i = 0$ (6.c)

B1B2: $\sum_{i=2}^{22} \Delta P_i = 0$ (6.d)

These equations can be simplified a little more. First, notice that all the terms in (6.a) are contained in (6.b), all the terms in (6.b) are contained in (6.c) and so on. Also, notice from (4) that some ΔP terms are equal to zero. By making all these substitutions, the short-circuit potential equations can finally be rewritten as:

M1M2: $\Delta P_{10} + \Delta P_{11} + \Delta P_{13} + \Delta P_{14} = 0$ (7.a)

N1N2: $\Delta P_{8} + \Delta P_{9} + \Delta P_{15} + \Delta P_{16} = 0$ (7.b)

A1A2: $\Delta P_{6} + \Delta P_{7} + \Delta P_{17} + \Delta P_{18} = 0$ (7.c)

B1B2: $\Delta P_{2} + \Delta P_{3} + \Delta P_{4} + \Delta P_{20} + \Delta P_{21} + \Delta P_{22} = 0$ (7.d)

In the short-circuit current equations the fact that currents can freely circulate trough the short-circuits is exploited. These equations are in fact current balance equations because for writing them the Kirchoff's Law of Currents is applied to each node in the tool. Notice from figure 2 that due to the short circuits, the 9 electrodes of the tool are reduced to 5 actual nodes. Notice also that, in addition to the currents supplied by the current sources, all currents injected and collected by the tool into and from the formation must be included in these equations. As it can be seen from figure 1, while electrodes A1 and A2 inject the bucking current Ia and A0 injects the survey current Io, electrodes B1 and B2 collect Ia+Io. On the other hand, the potential monitoring electrodes M1, M2, N1, N2 do not inject or collect any current. Then, by using these facts and applying Kirchoff's Law of Currents to each of the 5 nodes in figure 2, the short-circuit current equations are obtained:

A0: $I_v - I_w - I_0 = 0$ (8.a)

M1M2: $I_u + I_w - I_v - I_x = 0$ (8.b)

N1N2: $I_t + I_x - I_u - I_y = 0$ (8.c)

A1A2: $I_s + I_y - I_t - I_z - I_a = 0$ (8.d)

B1B2: $I_0 + I_a + I_z - I_s = 0$ (8.e)

where equation (8.a) is actually a linear combination of the others. To prove this, let us solve (8.e) for Io+Ia and replace it into (8.d), then solve (8.d) for Io and replace it into (8.c) and so on.

By doing that, it can be seen that (8.a) reduces to the trivial equality Io=Io; so (8) can be rewritten in terms of only four equations as follows:

$$I_s - I_z = I_0 + I_a \tag{9.a}$$

$$I_t - I_y = I_0 \tag{9.b}$$

$$I_u - I_x = I_0 \tag{9.c}$$

$$I_v - I_w = I_0 \tag{9.d}$$

Finally, in the control condition equations, the restrictions required for making the tool to operate in its normal focused condition are imposed. In this case those restrictions are imposed just by equating the potential between electrodes M1 and N1 to zero. Alternatively, the potential between M2 and N2 can be done zero.

M1N1: $\Delta P_8 + \Delta P_9 = 0$ (10)

or, M2N2: $\Delta P_{15} + \Delta P_{16} = 0$ (11)

Notice that either (9) or (10) must be used but not both of them. In fact, by including both of them no additional information is added to the set of equations; notice that the combination of (10) and (11) leads to (7.b).

At this point we already have all the required equations: 23 Method of Moment equations (the 5 electrode equations given in (4) and the 18 insulator equations given in (5)) and 9 System equations (the 4 short-circuit potential equations given in (7), the 4 short-circuit current equations given in (9) and the control condition equation given by either (10) or (11)). That accounts for a total of 32 linearly independent equations.

SIMPLIFICATION OF THE SET OF EQUATIONS

From the previous section, we obtained a set of 32 equations with 32 unknowns that can be solved, but programming a set like that can be quite involving. For this reason let us first attempt to simplify it a little more by using some algebraic manipulations and let us see if it can be written in a more compact form.

First of all, notice that (5) can be used to eliminate the unknown current source intensities in (9). By doing so and rearranging some terms, (9) can be rewritten as follows:

$$I_{20} = I_2 - I_a - I_0 \tag{12.a}$$

$$I_{17} = I_6 - I_0 \tag{12.b}$$

$$I_{15} = I_8 - I_0 \tag{12.c}$$

$$I_{13} = I_{10} - I_0 \tag{12.d}$$

Next, (5) and (12) can be used to collect some terms in the potential difference expressions. For example, by replacing (5) and (12) into (4.a) the following expression for ΔP_1 is obtained:

$$\begin{aligned}\Delta P_1 = {} & I_1\,\Delta R_0 + I_2\left(\sum_{i=1}^{3}\Delta R_i + \sum_{i=19}^{21}\Delta R_i\right) + I_5\,\Delta R_4 + I_6\left(\sum_{i=5}^{6}\Delta R_i + \sum_{i=16}^{17}\Delta R_i\right)\\ & + I_8\left(\sum_{i=7}^{8}\Delta R_i + \sum_{i=14}^{15}\Delta R_i\right) + I_{10}\left(\sum_{i=9}^{10}\Delta R_i + \sum_{i=12}^{13}\Delta R_i\right) + I_{12}\,\Delta R_{11}\\ & + I_{19}\,\Delta R_{18} + I_{23}\,\Delta R_{22} - I_0\left(\sum_{i=12}^{17}\Delta R_i + \sum_{i=19}^{21}\Delta R_i\right) - I_a\sum_{i=19}^{21}\Delta R_i\end{aligned} \tag{13}$$

In the same way, similar expressions for all the ΔP_n (n =1, 2, ... 23) can be written. And, what is much better, a general expression can be written as follows:

$$\begin{aligned}\Delta P_n = {} & I_1\,\Delta R_{1-n} + I_2\left(\sum_{i=2-n}^{4-n}\Delta R_i + \sum_{i=20-n}^{22-n}\Delta R_i\right) + I_5\,\Delta R_{5-n} + I_6\left(\sum_{i=6-n}^{7-n}\Delta R_i + \sum_{i=17-n}^{18-n}\Delta R_i\right)\\ & + I_8\left(\sum_{i=8-n}^{9-n}\Delta R_i + \sum_{i=15-n}^{16-n}\Delta R_i\right) + I_{10}\left(\sum_{i=10-n}^{11-n}\Delta R_i + \sum_{i=13-n}^{14-n}\Delta R_i\right) + I_{12}\,\Delta R_{12-n}\\ & + I_{19}\,\Delta R_{19-n} + I_{23}\,\Delta R_{23-n} - I_0\left(\sum_{i=13-n}^{18-n}\Delta R_i + \sum_{i=20-n}^{22-n}\Delta R_i\right) - I_a\sum_{i=20-n}^{22-n}\Delta R_i\end{aligned} \tag{14}$$

for n = 1, 2, 3, ..., 23.

Notice that equation (14) provides an expression for the potential difference between the centers of any two consecutive segments along the tool surface. Also, as it can be seen form (14), the total number of unknowns has been reduced to 10. Then, only 10 linearly independent equations are required now; and those are the 5 electrode equations, the 4 short-circuit potential equations and the control condition equation. By recalling the fact mentioned before that (7.b) was actually a linear combination of (10) and (11), the set of equations can be rewritten as follows:

$$\Delta P_1 = \Delta P_5 = \Delta P_{12} = \Delta P_{19} = \Delta P_{23} = 0 \tag{15.a}$$

$$\Delta P_8 + \Delta P_9 = \Delta P_{15} + \Delta P_{16} = 0 \tag{15.b}$$

$$\Delta P_{10} + \Delta P_{11} + \Delta P_{13} + \Delta P_{14} = 0 \tag{15.c}$$

$$\Delta P_6 + \Delta P_7 + \Delta P_{17} + \Delta P_{18} = 0 \tag{15.d}$$

$$\Delta P_2 + \Delta P_3 + \Delta P_4 + \Delta P_{20} + \Delta P_{21} + \Delta P_{22} = 0 \tag{15.e}$$

where the ΔP's are given by (14).

Finally, (15) can be written in matrix form as:

$$[\Delta R]\,\bar{I} = \bar{b} \tag{16}$$

where $[\Delta R]$ is a 10x10 matrix which elements are given by combination of ΔR's according to (14) and (15), $\bar{I}$ is the unknown column vector $\left[I_1\ I_2\ I_5\ I_6\ I_8\ I_{10}\ I_{12}\ I_{19}\ I_{23}\ I_a\right]^T$ and $\bar{b}$ is a column vector which elements are defined by the survey current Io times a combination of ΔR's.

At this point, programming the set of equation is very simple. The unknown vector can be computed by directly inverting the $[\Delta R]$ matrix.

THE SYMMETRIC TOOL CASE

Additional simplifications can be performed to the set of equations by assuming that certain symmetry conditions are held.

Although it is not always true in the practice, some times the laterolog system happens to be symmetric about its central electrode A0 (see figure 1). That means that the lower side of the tool is a mirror image of the upper side of the tool. This is going to be referred as the tool symmetry condition. In addition to that, if the borehole and formation parameters are assumed to be constant in the vertical direction, which is going to be referred as the formation symmetry condition, then the symmetry can be exploited by using the Bisection theorem.

When both the tool and the formation symmetry conditions are met, the following assertions happen to be true:

$$I_n = -I_{24-n} \quad \text{for } n = 1,2,...,12 \tag{17.a}$$

$$\Delta P_n = -\Delta P_{24-n} \quad \text{for } n = 1,2,...,12 \tag{17.b}$$

$$I_t = -I_y = I_u = -I_x = I_v = -I_w = \frac{I_0}{2} \tag{17.c}$$

$$I_s = -I_z = \frac{I_0 + I_a}{2} \tag{17.d}$$

Although the equations in (17) specifically refers to the model tool presented in figure 2, similar equations can be easily written for any symmetric tool configuration. The basic idea is to make currents and potential differences to be symmetric about the center of the tool. It is important to notice that the symmetry conditions make the presence of the short circuits in the LLs system to be redundant; that is, that they can be removed without altering the currents and the potentials. In fact, as it is going to see next, the use of (17) reduces the short circuit potential equations to trivial equalities.

By replacing (17) into (15) many of the equations become trivial and the set is then reduced to the following set of three linearly independent equations:

$$\Delta P_1 = 0 \tag{18.a}$$

$$\Delta P_5 = 0 \tag{18.b}$$

$$\Delta P_8 + \Delta P_9 = 0 \tag{18.c}$$

where, after replacing (17) into (14), the ΔP's are given by:

$$\Delta P_n = I_1\left(\Delta R_{1-n} - \Delta R_{23-n}\right) + I_5\left(\Delta R_{5-n} - \Delta R_{19-n}\right)$$
$$+\frac{I_a}{2}\left(\sum_{i=2-n}^{4-n}\Delta R_i - \sum_{i=20-n}^{22-n}\Delta R_i\right) + \frac{I_0}{2}\left(\sum_{i=2-n}^{4-n}\Delta R_i + \sum_{i=6-n}^{11-n}\Delta R_i - \sum_{i=13-n}^{18-n}\Delta R_i - \sum_{i=20-n}^{22-n}\Delta R_i\right) \quad (19)$$

for n = 1, 2, 3, ..., 23.

Notice from (19) that the number of unknowns have also been reduced to three. Then, by substituting (19) into (18) the set can be finally written in matrix form as in (16), with:

$$[\Delta R] = \begin{bmatrix} (\Delta R_0 - \Delta R_{22}) & (\Delta R_4 - \Delta R_{18}) & \frac{1}{2}\left(\sum_{i=1}^{3}\Delta R_i - \sum_{i=19}^{21}\Delta R_i\right) \\ (\Delta R_{-4} - \Delta R_{18}) & (\Delta R_0 - \Delta R_{14}) & \frac{1}{2}\left(\sum_{i=-3}^{-1}\Delta R_i - \sum_{i=15}^{17}\Delta R_i\right) \\ \left(\sum_{i=-8}^{-7}\Delta R_i - \sum_{i=14}^{15}\Delta R_i\right) & \left(\sum_{i=-4}^{-3}\Delta R_i - \sum_{i=10}^{11}\Delta R_i\right) & \frac{1}{2}\left(\sum_{i=-6;-7}^{-4;-5}\Delta R_i - \sum_{i=12;11}^{14;13}\Delta R_i\right) \end{bmatrix} \quad (20.a)$$

$$\bar{b} = -\frac{I_0}{2}\begin{bmatrix} \left(\sum_{i=1}^{3}\Delta R_i + \sum_{i=5}^{10}\Delta R_i - \sum_{i=12}^{17}\Delta R_i - \sum_{i=19}^{21}\Delta R_i\right) \\ \left(\sum_{i=-3}^{-1}\Delta R_i + \sum_{i=1}^{6}\Delta R_i - \sum_{i=8}^{13}\Delta R_i - \sum_{i=15}^{17}\Delta R_i\right) \\ \left(\sum_{i=-6;-7}^{-4;-5}\Delta R_i + \sum_{i=-2;-3}^{3;2}\Delta R_i - \sum_{i=5;4}^{10;9}\Delta R_i - \sum_{i=12;11}^{14;13}\Delta R_i\right) \end{bmatrix} \quad (20.b)$$

and, $\bar{I} = \begin{bmatrix} I_1 & I_5 & I_a \end{bmatrix}^T$ (20.c)

where the double indexed summations must be interpreted as follows:

$$\sum_{i=a;c}^{b;d}\Delta R_i = \sum_{i=a}^{b}\Delta R_i + \sum_{i=c}^{d}\Delta R_i \quad (21)$$

THE GENERAL SYMMETRIC TOOL CASE

The set of equations described by (18) and (19) refers to the particular case of the model tool presented in figure 2. Nevertheless, by looking carefully to the currents and the limits of the summations at every term in (19), it can be noticed that there exists a close relationship between those parameters and the tool configuration. Also, it can be noticed that the equations in (18) only make reference to the potentials over the electrodes and the control condition in one half of the tool. By exploiting those facts, we will be able to write the equations for an arbitrary LLs system which satisfies both of the symmetry conditions described before.

Figure 3 presents the specifications of an arbitrary symmetric LLs system of N segments in length. Notice that the short circuits have been omitted in figure 3. That is because, as it was mentioned before, when the symmetry conditions are held, the short circuits do not alter the system operation. Also, all the distances defined in figure 3 are indicated in numbers of segment lengths. So, for example, the size of electrode A1 is (NAE-NAS) segments.

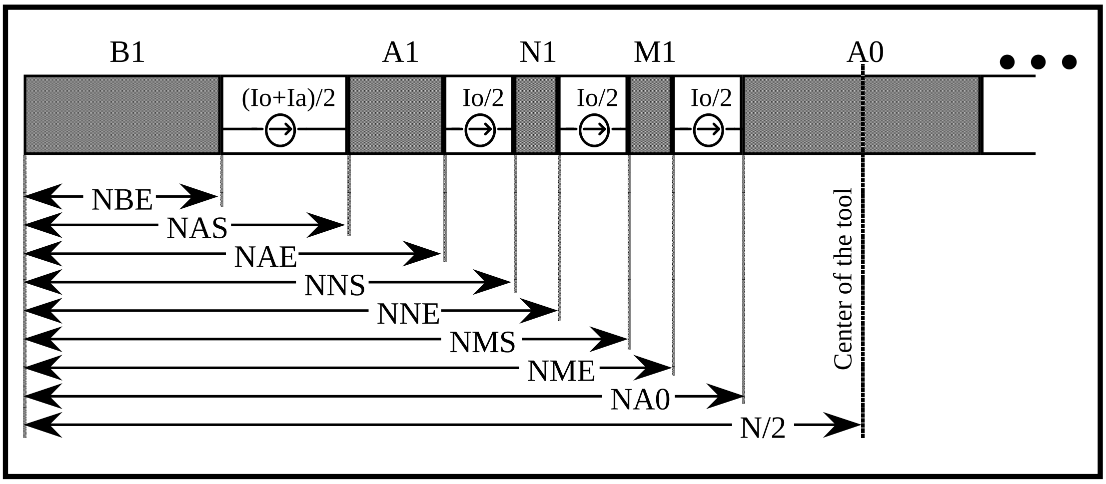

Figure 3: General Symmetric Tool Case.

By using the notation presented in figure 3 and the observations mentioned before, the set of equations for the general symmetric tool case can be written as follows:

$\Delta P_k = 0$ for $1 \le k < NBE$ (22.a)

$\Delta P_k = 0$ for $NAS < k < NAE$ (22.b)

$\Delta P_k = 0$ for $NNS < k < NNE$ (22.c)

$\Delta P_k = 0$ for $NMS < k < NME$ (22.d)

$\Delta P_k = 0$ for $NA0 < k < N/2$ (22.e)

$$\sum_{i=NNE}^{NMS} \Delta P_i = 0 \tag{22.f}$$

where the ΔP's are given by:

$$\Delta P_n = \sum_{i=1}^{NBE-1} \left(\Delta R_{i-n} - \Delta R_{N-i-n}\right) I_i + \sum_{i=NAS+1}^{NAE-1} \left(\Delta R_{i-n} - \Delta R_{N-i-n}\right) I_i + \sum_{i=NNS+1}^{NNE-1} \left(\Delta R_{i-n} - \Delta R_{N-i-n}\right) I_i$$

$$+ \sum_{i=NMS+1}^{NME-1} \left(\Delta R_{i-n} - \Delta R_{N-i-n}\right) I_i + \sum_{i=NA0+1}^{N/2-1} \left(\Delta R_{i-n} - \Delta R_{N-i-n}\right) I_i + \left(\sum_{i=NBE}^{NAS} \Delta R_{i-n} - \sum_{i=NAS}^{NBE} \Delta R_{N-i-n} \right) \frac{I_a}{2}$$

$$+ \left(\sum_{i=NBE}^{NAS} \Delta R_{i-n} + \sum_{i=NAE}^{NNS} \Delta R_{i-n} + \sum_{i=NNE}^{NMS} \Delta R_{i-n} + \sum_{i=NME}^{NA0} \Delta R_{i-n} \right) \frac{I_0}{2}$$

$$- \left(\sum_{i=NA0}^{NME} \Delta R_{N-i-n} + \sum_{i=NMS}^{NNE} \Delta R_{N-i-n} + \sum_{i=NNS}^{NAE} \Delta R_{N-i-n} + \sum_{i=NAS}^{NBE} \Delta R_{N-i-n} \right) \frac{I_0}{2} \tag{23}$$

for n = 1, 2, 3, ..., N-1.

It can be easily seen from (22) and (23) that the number of equations is equal to the number of unknowns. Then, by replacing (23) into (22) the system can be rewritten in matrix form an solved by a direct matrix inversion.

CONCLUSIONS

In the present report, the basic methodology for modeling a laterolog system has been discussed. Although the particular case of the LLs system was studied, the results obtained here can be easily extrapolated for the analysis of other type of laterolog systems. In fact, the five types of equations described here are applicable to any class of laterolog tool.

As it could be seen, by exploiting the symmetry properties of both the tool and the formation, the linear algebra problem can be greatly simplified. In general, the number of equations will be reduced to N/2-2; where N is the original number of equations without taking advantage of the symmetry conditions. Although the symmetry conditions not always happen in the practice, the use of symmetry when it is possible will notably reduce the time of computation.

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