Modeling of Generic Logging Devices

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INTRODUCTION

This report describes how to solve the direct version of the time harmonic field electric logging problem for a generic logging device. As it is described in [1], the electromagnetic equations are solved only for the basic current element and the result is then used to represent the logging device as a linear combination of a large number of current elements. For this reason, most of the attention in this report will be directed to the linear algebra problem.

A GENERIC LOGGING TOOL EXAMPLE

Although the procedure that will be presented here is properly suited for solving almost any tool configuration; it is for practical reasons and in order to make the definitions more understandable that the methodology will be illustrated by using a representative logging tool as an example. This representative tool is depicted in figure 1.

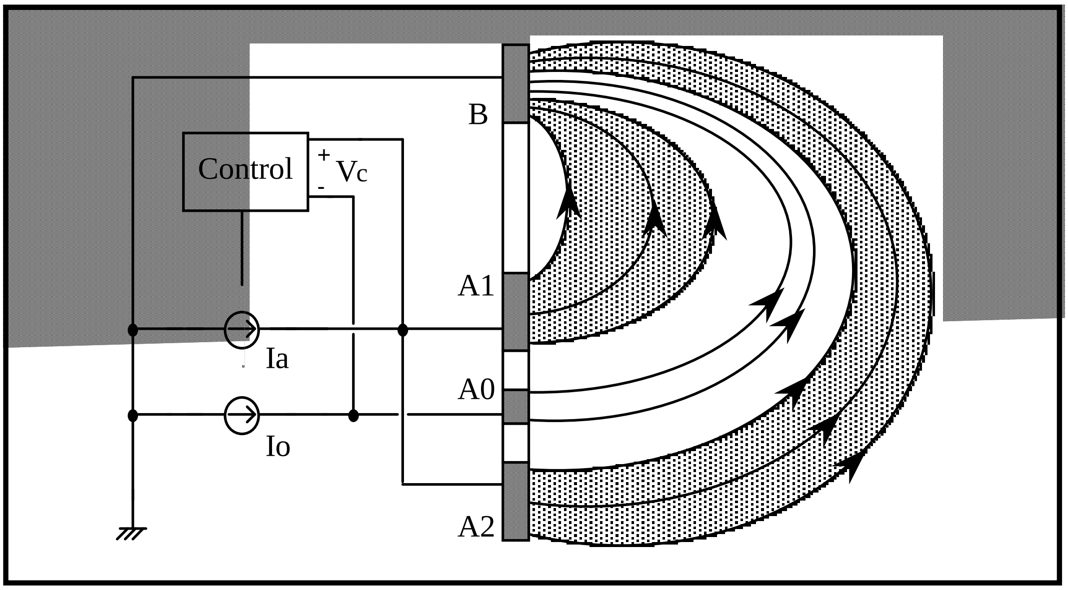

Figure 1: Generic tool example to be used for illustrative purposes.

The left side of figure 1 shows the electronics of the generic tool to be considered, while the right side illustrates the resulting current distribution in a homogeneous formation. As it can be seen from figure 1, the generic tool to be considered is composed by one current return electrode B and three current injector electrodes: the survey electrode A0, which injects the survey current Io into the formation; and the auxiliary or bucking electrodes A1 and A2, which are short-circuited to each other and inject the bucking current Ia. The intensity of the bucking current is adjusted by a control system so that certain potential condition Vc is achieved between the survey and the bucking electrodes. All the injected current Ia+Io is recollected by the current return electrode at the top of the tool after circulating throughout the earthen formation.

Although the particular configuration of a logging tool such as the one shown in figure 1 may not occur to be a real logging device, it was selected for explanatory purposes because, in addition to its relative simplicity, it includes all the elements present in the more complex logging systems.

TOOL MODEL AND SYSTEM UNKNOWNS

As it was mentioned before, the logging tool is modeled by considering the linear combination of a large amount of basic current elements. This procedure is known as the method of moments [2]. Figure 2 presents the appropriate model for the tool described in figure 1.

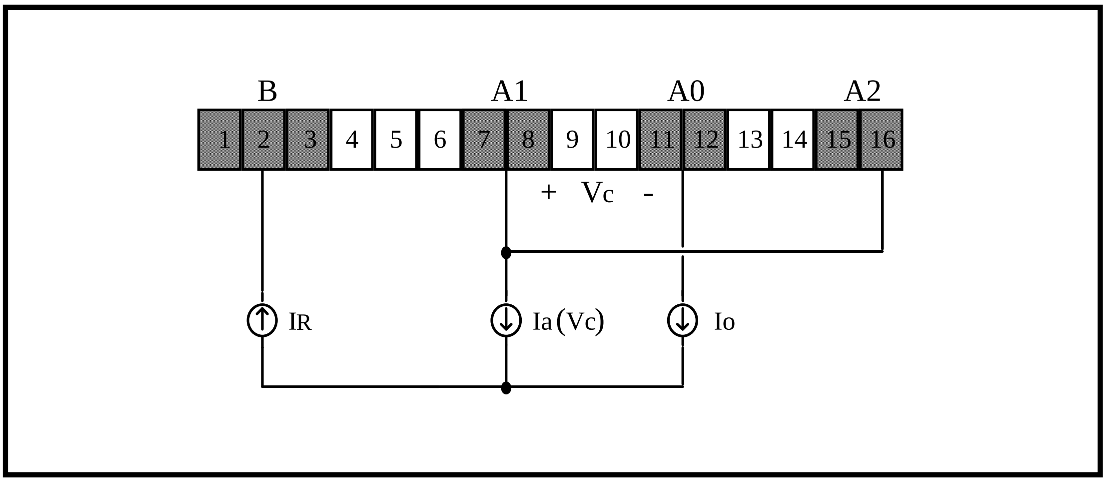

Figure 2: Model of the generic tool to be considered.

As it can be seen from figure 2, the size of the tool is 16 segment lengths. The basic current element, described in [1], has a total length of two segment lengths. So, only 15 current elements are required for representing the logging tool. Those current elements are going to be enumerated as 1 to 15 from left to right and their current strengths, which are unknowns to be solved for, will be denoted as I_1 to I_{15} respectively.

It can be noticed also that the intensity of the source representing the bucking current Ia depends on the potential condition V_C defined between the electrodes A1 and A0. So, the bucking current Ia will be other unknown in the system of equations. On the other hand, the intensity of the survey current Io must be known and different than zero.

Finally, it can be seen the presence of a current source I_R, which represents the current returning to the tool through the return electrode B. This current will also constitute an unknown. At this point, it can seem useless to consider the return current as an unknown since, as it can be seen from figure 2, it is just the addition of the bucking and the survey current; however, as it will be seen later, this consideration helps to write the system of equations in a more simple and standardized way.

Then, summarizing, the system represented in figure 2 has a total of 17 unknowns, which are: the 15 element current strengths, the bucking current Ia and the return current I_R. Therefore, 17 independent equations are required. As it will be seen next, two different kind of equations are involved, the method of moment equations and the system equations.

METHOD OF MOMENT EQUATIONS

The method of moment equations are those related to the fact that the tool is being modeled as a linear combination of current elements. As each current element is responsible for one equation, the total number of method of moment equations will be equal to the number of current elements in the model. Two different types of method of moment equations can be distinguished, electrode equations and insulator equations; and, as it is implied by their connotations, the type of equation related to an specific current element will depend on whether that element is located in an electrode or in an insulated region.

In the electrode equations, the potential differences between the centers of consecutive segments are equated to zero. That is because electrodes are supposed to be made out of perfect conductors. The number of equations in each electrode will depend on the size of the electrode. For an electrode of n segment lengths, there will be n-1 independent equations. So, a total of five electrode equations can be written for the tool represented in figure 2; one of which would be:

$$\Delta P_1 = I_1 \Delta R_0 + I_2 \Delta R_1 + I_3 \Delta R_2 + ... + I_{15} \Delta R_{14} = 0 \qquad (1)$$

where ΔP_1 is the potential difference between the centers of segments 1 and 2, the I's are the curr-ent strengths of the current elements and the ΔR's are the quantities, defined by (49) in [2], that represent the response of the basic current element.

What equation (1) simply states is that the potential difference between the centers of segments 1 and 2 is equal to zero, which is true since those two points are over the same electrode, see figure 2. Notice also from (1) that each term in the summation represents the contribution of each particular current element to the whole potential.

In general, the term $I_n \Delta R_k$ will be the contribution of the nth current element to the potential defined in a segment located at a distance of k segment lengths from it. In this way, the potential difference between the centers of segments n and n+1 can be defined as follows:

$$\Delta P_n = \sum_{i=1}^{N-1} \Delta R_{i-n} I_i \qquad (2)$$

where $\Delta R_{-n} = \Delta R_n$ [2], and N is the total number of segments in the tool model. For the example in figure 2, N=16.

Then, the five electrode equations for the tool presented in figure 2 are:

B: $\Delta P_1 = 0$ (3.a)

B: $\Delta P_2 = 0$ (3.b)

A1: $\Delta P_7 = 0$ (3.c)

A0: $\Delta P_{11} = 0$ (3.d)

A2: $\Delta P_{15} = 0$ (3.e)

In the insulator equations, the current strengths of the current elements in the same insulated region are equated to each other. That is because the tool surface in between electrodes is supposed to be made out of a perfect insulator, so no radial current flow exists. In fact, in the insulated regions, all current circulates along the tool's axial direction. The number of equations in each insulated region depends on the size of the region. For an insulated region of n segment lengths, there will be n+1 independent equations. For the example under consideration, the following 7 insulator equations can be written:

B_A1: $I_3 = I_4 = I_5 = I_6$ (4.a)

A1_A0: $I_8 = I_9 = I_{10}$ (4.b)

A0_A2: $I_{12} = I_{13} = I_{14}$ (4.c)

At this point, we have 12 equations out of the required 17.

SYSTEM EQUATIONS

The system equations are those related to the specific system configuration and its operating conditions. Again, two different types of system equations are to be considered, they are the potential condition equations and the nodal current equations.

In the potential condition equations, any potential restriction imposed to the tool is considered. Basically, two different kind of situations can lead to such a restriction. The first is the case of short-circuited electrodes. In this case, the potential difference between two electrodes is forced to be zero because of the existence of the short-circuit. This particular type of equations is going to be referred later as short-circuit potential equations. The logging tool under consideration exemplifies this situation with its bucking electrodes A1 and A2; the resulting equation follows:

$$\sum_{i=8}^{14} \Delta P_i = 0 \qquad (5)$$

The second situation arises when a bucking current is constrained by the occurrence of some defined potential condition between two independent electrodes in the tool (here, the term 'independent' makes reference to two electrodes that are no short-circuited to each other). Again, the tool presented in figure 2 exemplifies this situation with the definition of the potential condition Vc between the bucking and the survey electrodes; the resulting equation follows:

$$\sum_{i=8}^{10} \Delta P_i = V_c \qquad (6)$$

As it will be seen later, any additional bucking or return current defined in the logging system will require an associated potential condition in order to maintain the system of equations solvable.

Finally, in the nodal current equations, a balance of currents in each of the independent nodes of the tool is done. Notice that the number of independent nodes in a logging tool depends on how many electrodes are short-circuited and is always equal to the total number of electrodes minus the total number of non-redundant short-circuits. In the case of the tool under consideration, there are only three independent nodes. Their three related equations are:

B: $I_3 - I_R = 0$ (7.1)

A1_A2: $-I_6 + I_8 - I_{14} + I_a = 0$ (7.2)

A0: $-I_{10} + I_{12} + I_0 = 0$ (7.3)

where those currents entering the nodes have been considered as negative, and those exiting the node as positive.

At this point the total 17 independent equations have been completed; five electrode equations (3), seven insulator equations (4), one short-circuit potential equation (5), one potential condition equation (6) and three nodal current equations (7).

SIMPLIFICATION OF THE SET OF EQUATIONS

From the previous section, a set of 17 equations with 17 unknowns was obtained for the logging system presented in figure 1. Although such a system can be computationally solved, a big simplification can still be made to it. The aim of such simplification is to use the insulator equations, given in (4), to reduce the number of unknowns. In fact, as the current strengths of all current elements in the same insulated region are equal, the number of actual unknown currents in the insulators can be reduced to the number of insulated regions in the logging device.

By replacing (4) into (2) the following expression results:

$$\Delta P_n = I_1\,\Delta R_{1-n} + I_2\,\Delta R_{2-n} + I_3 \sum_{i=3}^{6} \Delta R_{i-n} + I_7\,\Delta R_{7-n}$$

$$+ I_8 \sum_{i=8}^{10} \Delta R_{i-n} + I_{11}\,\Delta R_{11-n} + I_{12} \sum_{i=12}^{14} \Delta R_{i-n} + I_{15}\,\Delta R_{15-n} \qquad (8)$$

which, after the respective substitutions, redefines equations (3), (5) and (6) in terms of the eight current strengths $I_1, I_2, I_3, I_7, I_8, I_{11}, I_{12}$ and I_{15}.

Also, by replacing (4) into (7), the nodal current equations are reduced to:

$$I_3 - I_R = 0 \qquad (9.a)$$

$$-I_3 + I_8 - I_{12} + I_a = 0 \qquad (9.b)$$

$$-I_8 + I_{12} + I_0 = 0 \qquad (9.c)$$

notice that the summation of the equations in (9) leads to the equation $I_R = I_a + I_0$; which, as it was mentioned before, can be easily deduced from figure 2. Now, it is made more evident the fact that the inclusion of this information would make the equations in (9) linearly dependent and would require the subsequent elimination of the linear dependence. So, the inclusion of the return current IR as an unknown allows the use of the entire set of nodal current equations (9).

Then, the original system of 17 equations with 17 unknowns has been reduced to a system of 10 equations, given by (3), (5), (6) and (9); and 10 unknowns, given by the 5 current strengths related to those current elements located in the electrodes (I_1, I_2, I_7, I_{11} and I_{15}), the 3 current strengths associated to the insulated regions (I_3, I_8 and I_{12}), the bucking current Ia and the return current IR.

MATRIX REPRESENTATION OF THE SYSTEM OF EQUATIONS

By introducing the following notation:

$$R_{n_a,n_b}^{i_a,i_b} = \sum_{n_a}^{n_b} \sum_{i_a}^{i_b} \Delta R_{i-n} \qquad (10.a)$$

it is possible to rewrite (8) in a more compact form as follows:

$$\Delta P_n = \overline{R}_n^{\,T}\, \overline{I}_{cs} \qquad (10.b)$$

where the vectors $\overline{R}_n$ and $\overline{I}_{cs}$ are given by:

$$\overline{R}_n^{\,T} = \left[R_n^1 \quad R_n^2 \quad R_n^7 \quad R_n^{11} \quad R_n^{15} \quad R_n^{3,6} \quad R_n^{8,10} \quad R_n^{12,14} \right] \qquad (10.b.1)$$

and $\overline{I}_{cs}^{\,T} = \left[I_1 \quad I_2 \quad I_7 \quad I_{11} \quad I_{15} \quad I_3 \quad I_8 \quad I_{12} \right]$ (10.b.2)

Then, it is also possible to rewrite the reduced system of equations as:

$$\overline{R}_1^{\,T}\, \overline{I}_{cs} = \overline{R}_2^{\,T}\, \overline{I}_{cs} = \overline{R}_7^{\,T}\, \overline{I}_{cs} = \overline{R}_{11}^{\,T}\, \overline{I}_{cs} = \overline{R}_{15}^{\,T}\, \overline{I}_{cs} = 0 \qquad (11.a)$$

$$\left(\sum_{i=8}^{14} \overline{R}_i \right)^T \overline{I}_{cs} = 0 \qquad (11.b)$$

$$\left(\sum_{i=8}^{10} \overline{R}_i \right)^T \overline{I}_{cs} = V_c \qquad (11.c)$$

$$I_3 - I_R = 0 \qquad (11.d)$$

$$-I_3 + I_8 - I_{12} + I_a = 0 \qquad (11.e)$$

$$-I_8 + I_{12} = -I_0 \qquad (11.f)$$

Notice that from (11), an immediate matrix representation of the system of equations easily follows, and it is given by:

$$[R]\, \overline{I} = \overline{b} \qquad (12.a)$$

where the matrix $[R]$ is the coefficient matrix of the system of equations, given by:

$$\begin{bmatrix} R_1^1 & R_1^2 & R_1^7 & R_1^{11} & R_1^{15} & R_1^{3,6} & R_1^{8,10} & R_1^{12,14} & 0 & 0 \\ R_2^1 & R_2^2 & R_2^7 & R_2^{11} & R_2^{15} & R_2^{3,6} & R_2^{8,10} & R_2^{12,14} & 0 & 0 \\ R_7^1 & R_7^2 & R_7^7 & R_7^{11} & R_7^{15} & R_7^{3,6} & R_7^{8,10} & R_7^{12,14} & 0 & 0 \\ R_{11}^1 & R_{11}^2 & R_{11}^7 & R_{11}^{11} & R_{11}^{15} & R_{11}^{3,6} & R_{11}^{8,10} & R_{11}^{12,14} & 0 & 0 \\ R_{15}^1 & R_{15}^2 & R_{15}^7 & R_{15}^{11} & R_{15}^{15} & R_{15}^{3,6} & R_{15}^{8,10} & R_{15}^{12,14} & 0 & 0 \\ R_{8,14}^1 & R_{8,14}^2 & R_{8,14}^7 & R_{8,14}^{11} & R_{8,14}^{15} & R_{8,14}^{3,6} & R_{8,14}^{8,10} & R_{8,14}^{12,14} & 0 & 0 \\ R_{8,10}^1 & R_{8,10}^2 & R_{8,10}^7 & R_{8,10}^{11} & R_{8,10}^{15} & R_{8,10}^{3,6} & R_{8,10}^{8,10} & R_{8,10}^{12,14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}; \qquad (12.b)$$

the vector $\bar{I}$ constitutes and extended version of the vector $\bar{I}_{cs}$, given in (10.b.2),

$$\bar{I}^T = \left[\bar{I}_{cs}^T \;|\; I_a \quad I_R \right]; \qquad (12.c)$$

and the vector $\bar{b}$ contains the independent parameters

$$\bar{b}^T = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad V_c \quad 0 \quad 0 \quad -I_0 \right]. \qquad (12.d)$$

Solution to the linear algebra problem is then achieved by direct inversion of the matrix $[R]$ and the computation of the unknown current vector $\bar{I}$. Finally, with all currents known, all the potential differences along the tool surface can be computed by means of (10).

GENERALIZATION OF THE METHOD

In the previous sections, the complete system of equations for the particular tool configuration presented in figure 1 has been determined. However, it has been done is a such standardized manner that the subsequent generalization to an arbitrary tool configuration will follow in a very natural way.

Firs of all, let us define some important tool parameters, which are presented in Table 1.

Table 1: Important tool parameter definitions.

Parameter	Definition
NE	Total number of electrodes.
Nshc	Number of non-redudant short-circuits.
Nn	Number of actual nodes.
N#s	First segment in electrode #.
N#e	Last segment in electrode #.
Npc	Number of potential conditions.
Nce	Total number of current elements at electrodes.
Nci	Total number of current elements at insulators.
Nui	Total number of unknown nodal currents.

Some important relationships between the parameters defined in Table 1 follow:

$$Nn = NE - Nshc \tag{13.1}$$

$$Nce = \sum_{k=1}^{NE} (Nke - Nks) \tag{13.2}$$

$$Nci = \sum_{k=1}^{NE-1} (N(k+1)s - Nke) \tag{13.3}$$

$$N = Nce + Nci + 1 \tag{13.4}$$

where N is the total number of segments in the tool model.

The total number of unknowns and equations for a given tool configuration can also be expressed in terms of the parameters in Table 1.

The total number of unknowns, as it was seen in previous sections, is given by the total amount of three different types of currents; they are, the current strengths associated to those current elements located at the electrodes (Nce), the current strengths of the elements at the insulators (which are reduced to (NE-1) unknowns by means of the insulator equations), and the unknown nodal currents (Nui) which include all bucking and return currents. Then, it can be written that:

$$Nunknowns = Nce + (NE-1) + Nui \tag{14}$$

On the other hand, the total number of equations that can be written for a given tool configuration is given by the electrode equations (Nce), the short-circuit potential equations

(Nshc), the potential condition equations (Npc) and the nodal current equations (Nn). Then, it follows that:

Nequations = Nce + Nshc + Npc + Nn (15.a)

or, by using (13.1),

Nequations = Nce + Npc + NE (15.b)

Notice from (14) and (15.b), that a necessary condition for the system to be solvable is that there must be one less potential condition than unknown nodal currents. That is, in the case of tools with only one return current, that the number of potential conditions must be equal to the number of bucking currents. Notice also that, depending upon the tool configuration, the short-circuit and the potential condition equations may not be required; which is not the case for the electrode and the nodal current equations that must always be present.

Once the agreement between the number of equations and unknowns in a given tool configuration is checked, its related set of equations can be written as follows:

A.- ELECTRODE EQUATIONS:

$$\Delta P_i = 0 \quad \forall \ i \in [\mathrm{Nks}, \mathrm{Nke}-1] \quad \text{for} \ \ k = 1, 2 \ldots \mathrm{NE} \tag{16.a}$$

B.- SHORT-CIRCUIT POTENTIAL EQUATIONS:

$$\sum_{i=\mathrm{Nke}}^{\mathrm{Nhs}-1} \Delta P_i = 0 \quad \forall \ \{k, h\} \ni \ k < h \ \wedge \ \mathrm{Ek} \leftrightarrow \mathrm{Eh} \tag{16.b}$$

where $\mathrm{Ek} \leftrightarrow \mathrm{Eh}$ means that there is a short-circuit between electrodes k and h.

C.- POTENTIAL CONDITION EQUATIONS:

$$\sum_{i=\mathrm{Nke}}^{\mathrm{Nhs}-1} \Delta P_i = V_{kh} \quad \forall \ \{k, h\} \ni \ k < h \ \wedge \ \mathrm{PC}(\mathrm{Nk}, \mathrm{Nh}) = V_{kh} \tag{16.c}$$

where $\mathrm{PC}(\mathrm{Nk}, \mathrm{Nh}) = V_{kh}$ means that the potential condition V_{kh} has been defined between nodes k and h.

D.- NODAL CURRENT EQUATIONS:

$$\sum_{\substack{\text{unknown}\\ \text{currents}}} I_{\text{exiting node } k} - \sum_{\substack{\text{unknown}\\ \text{currents}}} I_{\text{entering node } k} = \sum_{\substack{\text{known}\\ \text{currents}}} I_{\text{entering node } k} - \sum_{\substack{\text{known}\\ \text{currents}}} I_{\text{exiting node } k} \qquad \text{for } \ k = 1, 2 \ldots Nn \qquad (16.d)$$

Where the potential differences ΔP_i's are computed by using the following generalization of (10):

$$\Delta P_i = \sum_{k=1}^{NE} \left(\sum_{m=Nks}^{Nke-1} I_m \ \Delta R_{m-i} \right) + \sum_{k=1}^{NE-1} \left(I_{Nke} \sum_{m=Nke}^{N(k+1)s \ -1} \Delta R_{m-i} \right) \qquad (16.e)$$

Again, the matrix representation given in (12.a) can be applied to (16). A detailed illustration of the matrix and vector distributions and sizes is provided in figure 3.

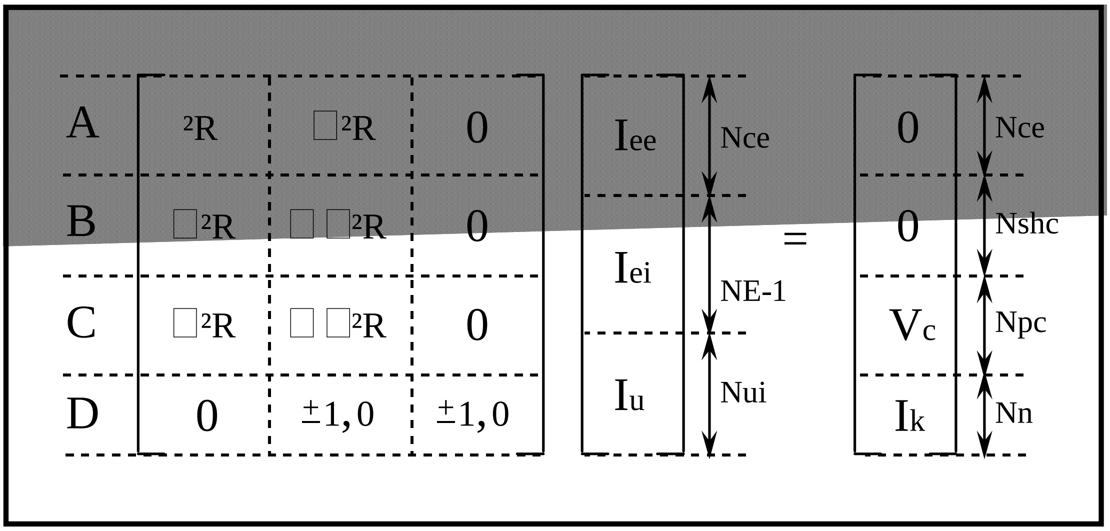

Figure 3: Composition of the matrix equation for a generic logging tool.

The use of the modeling procedure discussed here not only allows to model generic logging tools, but also to model different configurations of the same instrument. This is particularly useful in the computation of circuit analogs for logging devices in a given formation [3]. This topic is briefly discussed in the following section.

CIRCUIT ANALOG COMPUTATION

As it is going to be seen next, it is possible to represent a logging device operating in a given earthen formation by means of an analogous circuit, in which the formation is simulated by an impedance network whose nodes represents the tool's electrodes. In such a representation, the set of impedances is totally determined by the geometry and conductivity distribution of the earthen formation, the physical arrangement of the tool's electrodes and the tool's frequency of operation. Notice, however, that it is independent of the tool's electronics.

In order to compute the impedance values for a given scenario (physical arrangement of electrodes, earthen formation and frequency of operation), the linear algebra problem has to be solved for a sequence of different tool configurations. In each of those configurations, the mutual impedances between one of the nodes and all others are measured. For simplicity, the impedance between nodes k and m is going to be referred as Z(k,m). Figure 4 illustrates the configuration that allows the computation of the impedances Z(1,2) and Z(1,3) for the example tool of figure 1.

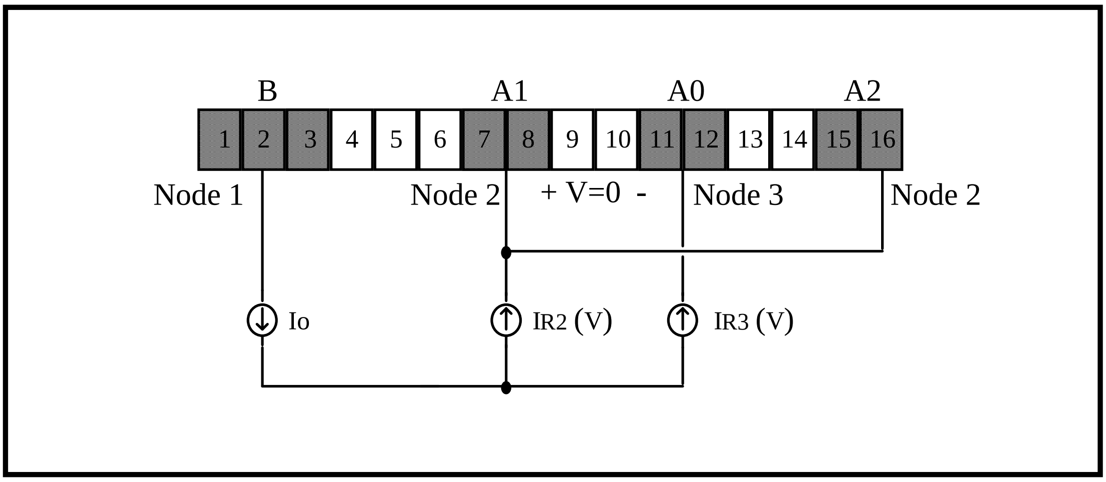

Figure 4: Configuration for measuring Z(1,2) and Z(1,3).

As it can be seen from figure 4, the four-electroded tool under consideration has three actual nodes. Node 1 is composed by the electrode B, node 2 by electrodes A1 and A2, and node 3 by

A0. Notice that in the particular configuration depicted in figure 1, node 1 injects the survey current Io which is recollected as two separated return currents by nodes 2 and 3. Additionally, a zero potential condition has been imposed between the later.

The situation presented in figure 4 is equivalent to that presented in figure 5, where the circuit analog is shown.

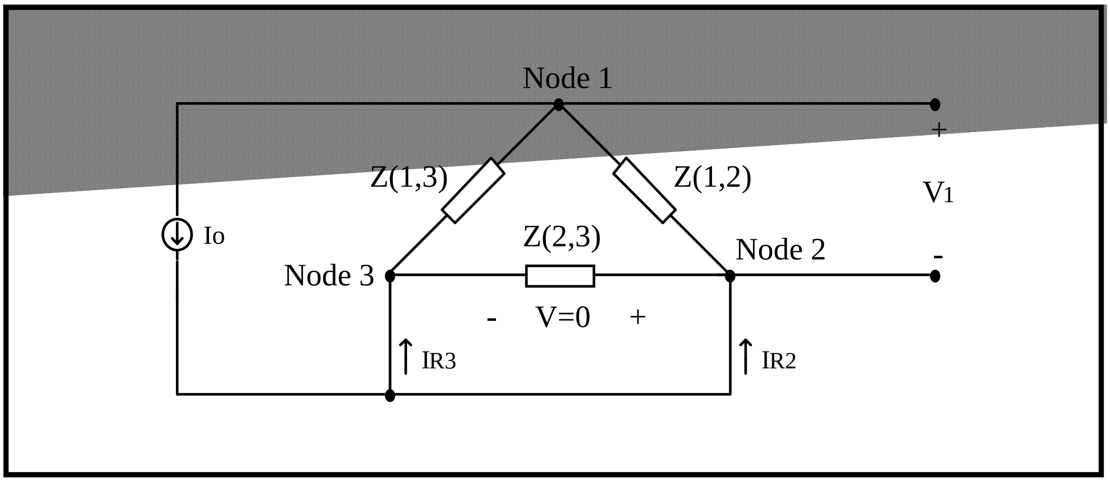

Figure 5: Circuit analog configuration for measuring Z(1,2) and Z(1,3).

From figure 5, it follows that :

$$Z(1,2) = -\frac{V_1}{I_{R2}} \qquad (17.a)$$

and $$Z(1,3) = -\frac{V_1}{I_{R3}} \qquad (17.b)$$

where, according to figure 4, V_1 can be computed (after solving the linear algebra problem) as:

$$V_1 = \sum_{i=3}^{6} \Delta P_i \qquad (17.c)$$

In a similar way, two additional configurations to compute Z(2,3) and Z(2,1), and Z(3,1) and Z(3,2) can be realized; and it can be verified that, as it is expected to be, Z(k,m) = Z(m,k) is always satisfied. Notice then, that in fact only two different configurations are required to fully determine the set of impedances in the circuit analog of the tool under consideration.

In general, for an arbitrary tool with Nn nodes, the impedance values for its circuit analog can be computed by solving the linear algebra problem for a total of Nn-1 different configurations, say i=1,2...Nn-1. The procedure related to the ith configuration is described next.

<u>PARAMETER DEFINITION FOR THE ith CONFIGURATION</u>

A.- The original tool's arrangement of electrodes and insulators is preserved.

B.- The original tool's short-circuits are maintained.

C.- All nodal currents are redefined as follows:

Node i: Injects survey current Io.

All other nodes: Collect return currents, being I_{Rk} the return current at node k. ($k \neq i$)

D.- The original potential conditions are ignored and Nn-2 new zero potential conditions are imposed such that all nodes, except node i, are forced to be at the same potential.

<u>IMPEDANCE COMPUTATION IN THE ith CONFIGURATION</u>

A.- Once the linear algebra problem is solved and all currents are known, the potential V_i (between the node i and any one of the others) is computed.

B.- The values of Z(i,k) for k = i+1, i+2 ... Nn are computed as $-V_i / I_{Rk}$.

CONCLUSIONS

In the present report, the methodology for solving the linear algebra problem for a generic logging device was developed. A detailed discussion about the different kind of equations that must be considered and its associated matrix representation was presented. Finally, the analog circuit representation of a logging system was briefly discussed.

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