

# The Direct Problem

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## INTRODUCTION

This report describes the methodology used to solve the direct version of the time harmonic field electric logging problem [1]. In the direct version of problem we are interested in compute the tool measurement given the earthen formation parameters and the tool configuration. Figure 1 illustrates a logging tool, its measurement and the kind of formation to be considered.

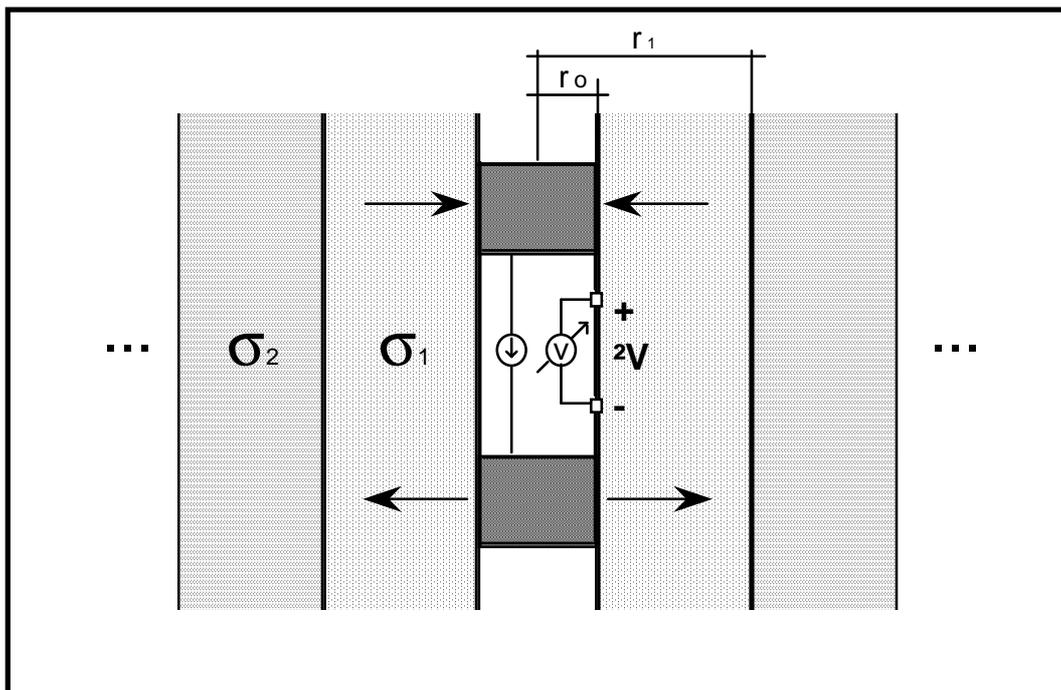


Figure 1: Logging tool inside a cylindrical earthen formation.

As it can be seen from Figure 1 the logging tool is assumed to be in a cylindrical borehole that is centered in a earthen formation composed of coaxial annular zones [1]. Each of the zones is considered to be homogeneous (with constant electric parameters). The logging tool presented in Figure 1 is a very simple one. It is composed by two electrodes, a current source and a measuring

device. The lower electrode injects the current into the formation. The current circulates through the formation and comes back to the tool through the upper electrode. The measuring device reads the potential difference generated between two points located along the tool surface. The idea of this report is to show how this potential difference can be numerically computed.

## THE METHOD OF MOMENTS AND THE BASIC CURRENT ELEMENT

The numerical computation of the potential difference shown in Figure 1 is going to be performed by using the Method of Moments [2]. The idea of this method is to obtain the final solution of the problem as a linear combination of elementary solutions. For this purpose, a basic current element is defined and the tool is then modeled by combining a large number of these elements. Figure 2 shows the basic current element and presents a very simple example of how a logging tool can be modeled.

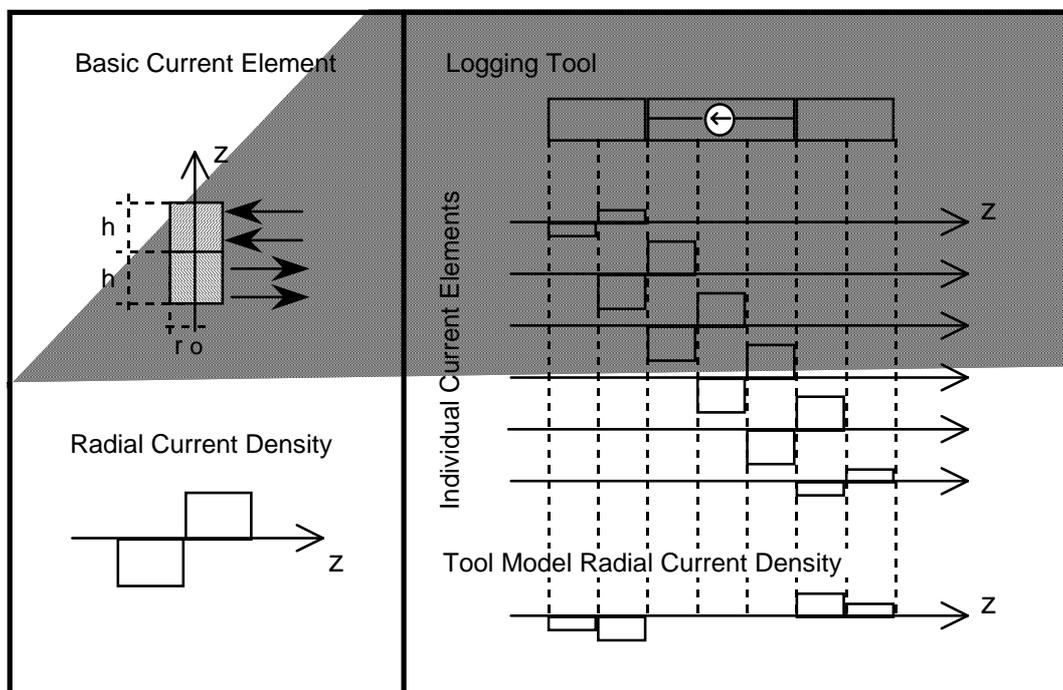


Figure 2: Basic Current Element and modeling of a simple Logging Tool.

As it can be seen from Figure 2 the basic current element is formed by the union of two electrodes which are assumed to have constant radial current densities. Both electrodes are cylinders of radius  $r_0$  and height  $h$ . The height  $h$  is going to be referred as the segment length. Another important parameter of the basic current element is its current strength, it is given by the total amount of current the element injects into the formation. It can be expressed in terms of the radial current density as:

$$I = \iint_{S_e} J_r \cdot ds \quad (1)$$

where  $J_r$  is the radial current density,  $ds = r_0 d\phi dz$  and  $S_e$  refers to the surface of the lower half of the current element.

It can be also seen from Figure 2 that in order to model a logging tool by using current elements, the electrodes length and the total length of the tool must be an integer multiple of the segment length  $h$ . In the practice, for a given logging tool, the segment length must be chosen in such a way that the length of the electrodes and the distance between them can be properly approximated with integer multiples of  $h$ . Notice also that for modeling a logging tool of total length  $Nh$ , only  $(Nh-1)$  current elements are required.

The solution of the direct problem can be performed in two clearly separated steps. In the first step, the current strengths of the  $(Nh-1)$  current elements used to model the tool are to be computed. In the second and last step, the potential difference measurement is to be computed. In this section, we are going to concentrate only in the first step. The second step is going to be discussed in the next section.

In order to compute the current strengths, the electromagnetic equations must be solved for the basic current element. A detailed analysis of this is presented in [1]. As a result of the electromagnetic analysis, the electric field produced by the current element is known. We are particularly interested in the  $z$  component of the field along the tool surface, that is at  $r = r_0$ . By integrating this component of the field along the  $z$  direction, the potential difference generated by

the current element along the tool surface can be computed. So, the potential difference between the centers of adjacent segments at a distance  $nh$  from the current element is given by:

$$\Delta P_n = \int_{\left(nh-\frac{h}{2}\right)}^{\left(nh+\frac{h}{2}\right)} E_z(r_0, z) dz \quad (2)$$

where  $E_z(r_0, z)$  is the  $z$  component of the electric field along the tool surface and  $h$  is the segment length.

Now, the quantity  $\Delta R_n$  is defined by normalizing (2) by the current strength [1].  $\Delta R_n$  is then given by:

$$\Delta R_n = \frac{1}{I} \int_{\left(nh-\frac{h}{2}\right)}^{\left(nh+\frac{h}{2}\right)} E_z(r_0, z) dz \quad (3)$$

where  $I$  is the current strength of the basic current element.

In this way, the potential difference generated by an arbitrary current element of current strength  $I_x$  between the centers of two adjacent segments at a distance  $nh$  can be expressed as:

$$\Delta P_n = I_x \Delta R_n \quad (4)$$

Figure 3 illustrates graphically the meaning of equation (4).

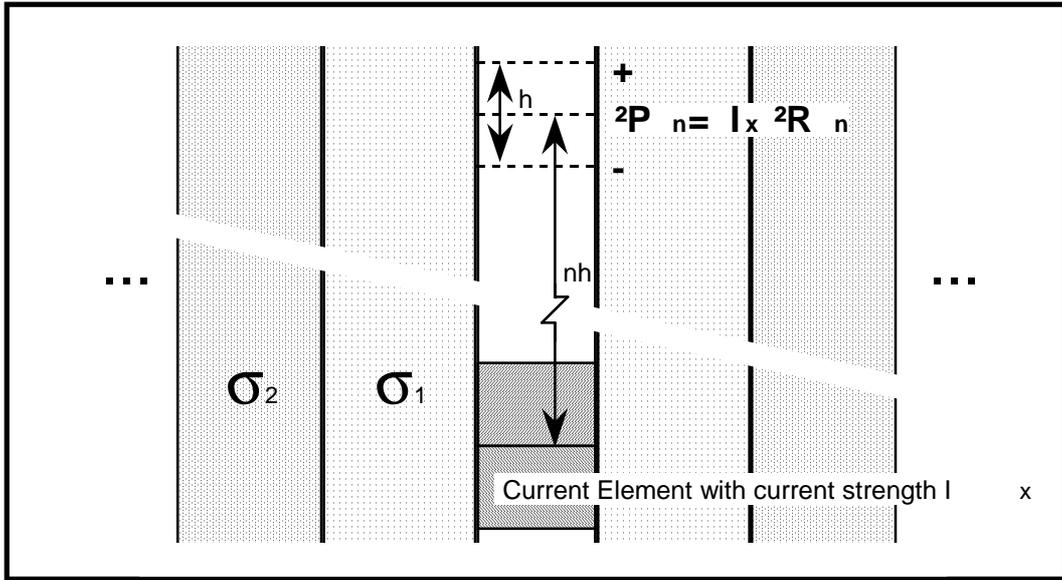


Figure 3: Potential Difference Produced by a Current Element.

By constructing a set of linear equations with the contributions of all the currents elements in the tool, it is possible to compute the current strengths of the current elements. For simplicity, let us consider the seven segment tool presented in Figure 2. Let us enumerate the six current elements from left to right as 1, 2 ... and 6; so that their current strengths are going to be referred as  $I_1$ ,  $I_2$ , ... and  $I_6$ .

Now, let us write the equations starting with the electrodes. As it can be noticed the potential difference between the centers of segments 1 and 2 is zero. That is because this potential is defined over the surface of the electrode, which is supposed to be a perfect conductor. In the same way, the potential between the centers of segments 6 and 7 is zero too. By adding up the contributions of all the current elements to the potentials in the electrodes we get the first two equations:

$$\text{for the left electrode: } 0 = I_1 \Delta R_0 + I_2 \Delta R_1 + I_3 \Delta R_2 + I_4 \Delta R_3 + I_5 \Delta R_4 + I_6 \Delta R_5 \quad (5)$$

$$\text{for the right electrode: } 0 = I_1 \Delta R_5 + I_2 \Delta R_4 + I_3 \Delta R_3 + I_4 \Delta R_2 + I_5 \Delta R_1 + I_6 \Delta R_0 \quad (6)$$

where the currents are the unknowns and the  $\Delta R$ 's are given by (3).

For the region in between the electrodes another consideration must be done. In this region the radial current flow is zero. That is because the tool surface in this region is suppose to be composed by a perfect insulator. In this region, all the current flows in the z direction from one electrode to the other. From this consideration it can be concluded that the current strengths of all current elements in this region must be the same, so that the radial current densities of the adjacent elements cancel each other and the total contribution to the tool radial current density would be zero. This is clearly illustrated in Figure 2. Then, three equations can be written from the insulated part of the tool; they are given by:

$$I_2 = I_3 = I_4 = I_5 \quad (7)$$

At this point, we have five equations and six unknowns. There is still one more equation missing. This last equation comes from the fact discussed above. In the insulated part of the tool the radial current flow is zero, which means that all the current flows in the z direction. This current is already known, and it is given by the intensity of the current source in the logging tool. Let us call it  $I_{\text{source}}$  or simply  $I_s$ . Then (7) becomes:

$$I_2 = I_3 = I_4 = I_5 = I_s \quad (8)$$

By replacing (8) into (5) and (6), a set of two equations with two unknowns is obtained. By arranging some terms it can be written as follows:

$$I_1 \Delta R_0 + I_6 \Delta R_5 = -I_s (\Delta R_1 + \Delta R_2 + \Delta R_3 + \Delta R_4) \quad (9.a)$$

$$I_1 \Delta R_5 + I_6 \Delta R_0 = -I_s (\Delta R_4 + \Delta R_3 + \Delta R_2 + \Delta R_1) \quad (9.b)$$

or, in matrix form:

$$\begin{bmatrix} \Delta R_0 & \Delta R_5 \\ \Delta R_5 & \Delta R_0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_6 \end{bmatrix} = \begin{bmatrix} -I_s (\Delta R_1 + \Delta R_2 + \Delta R_3 + \Delta R_4) \\ -I_s (\Delta R_4 + \Delta R_3 + \Delta R_2 + \Delta R_1) \end{bmatrix} \quad (10)$$

In the general case, the set of equations represented in (10) is going to look as follows:

$$[\Delta R] \bar{I} = \bar{b} \quad (11)$$

where  $[\Delta R]$  is a square matrix, which elements are all computed from (3);  $\bar{I}$  is the unknown column vector, which elements correspond to the current strengths of the current elements in the tool electrodes; and  $\bar{b}$  is a column vector of known coefficients that are related to the intensity  $I_s$  of the current source of the logging tool.

The unknown current strengths can be computed now by inverting the matrix  $[\Delta R]$  and solving equation (10) for  $\bar{I}$ .

$$\bar{I} = [\Delta R]^{-1} \bar{b} \quad (12)$$

### COMPUTATION OF THE TOOL MEASUREMENT

Once the current strengths of all the current elements in the tool are known, the potential difference measurement can be easily computed.

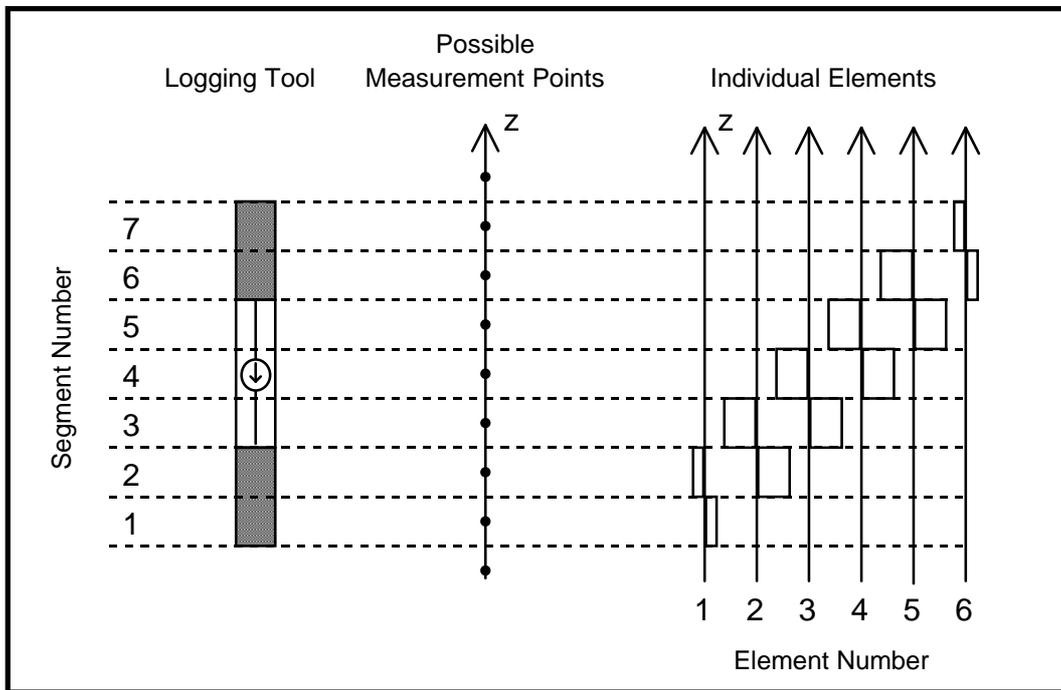


Figure 4: Logging Tool and its Possible Measurement Points.

As there was a restriction for the tool and the electrodes lengths, given by the segment length size  $h$ ; there is also a restriction for the points in between which the potential difference can be measured. In this case, the measurement must be done between segment central points. Figure 4 illustrates this restriction. For simplicity, the same tool presented in Figure 2 is considered.

Let us compute, as an example, the potential difference between the central points of segments 5 and 3 in Figure 4. According to (4), the contribution of the first current element to that potential would be  $I_1 \Delta R_2 + I_1 \Delta R_3$ . By adding up all the contributions from all the current elements, the potential difference between the centers of segments 5 and 3 would be given by:

$$\Delta P_{S_5, S_3} = (I_1 + I_6)(\Delta R_2 + \Delta R_3) + (I_2 + I_5)(\Delta R_1 + \Delta R_2) + (I_3 + I_4)(\Delta R_0 + \Delta R_1) \quad (13)$$

## CONCLUSIONS

Although the algebraic solution of the direct problem by using the method of moments is very straight forward, there is still a very difficult task to be discussed. That is the numerical computation of the  $\Delta R$ 's defined in (4). A detailed analysis of the electromagnetic equations and the procedure to obtain a mathematical expression for the  $\Delta R$ 's is presented in [1]. There, it can be seen that the computation of the  $\Delta R$ 's involves the numerical evaluation of a Fourier Transform integral that constitutes the actual challenge of the direct problem solution. The methodology used for the numerical evaluation of (4) will be discussed in future reports.

## REFERENCES

- [1] Bostick, F.; Smith, H. (1994), Propagation Effects in Electric Logging.  
University of Texas at Austin.

- [2] Balanis, C. (1989), Advanced Engineering Electromagnetics. John Wiley & Sons.