Seismic Inversion with an Infinite Impulse Response Convolutional Model

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Summary

A seismic inversion algorithm based on an infinite impulse response (*IIR*) convolutional model is proposed. In the presented technique, which allows for modeling primary and multiple reflections, the subsurface response is represented by means of a normalized lattice filter, and the synthetic seismic traces are computed as the convolution between the seismic wavelet and the filter impulse response. The inversion algorithm uses a hybrid optimization scheme that is based on genetic algorithms to perform the global search and a simplex method to perform the local search.

Introduction

In a convolutional model, synthetic seismic traces are defined to be the convolution between a seismic wavelet and a reflectivity series that represents the earthen formation response (Waters, 1981). Traditionally, this response is expressed in terms of a finite length time series. In the convolutional model presented in this paper, a normalized lattice filter (Oppenheim & Schafer, 1975) is used to represent the earthen response; so the subsurface is characterized by an infinite impulse response (*IIR*) instead of a finite impulse response (*FIR*). The advantage of using such a kind of filters for representing the formation is that it allows the modeling of primary reflections as well as multiple reflections. This can be of great utility in the study of zones where multiples can represent a problem.

The inverse modeling algorithm implemented is based on an hybrid optimization scheme (Chunduru, 1996). First, a global search is performed by a genetic algorithm, which is followed by a local search performed by a simplex method. In the global search, the non linear capabilities of genetic algorithms are exploited in order to handle the non linear character of the objective functions used. The inversion is performed in a sequence of alternating steps in which two different types of model parameters (reflection coefficients and time delays) are adjusted independently.

First, the proposed *IIR* convolutional model is described in detail. Then, the inversion algorithm is discussed. And finally, two synthetic examples of seismic inversion with the *IIR* convolutional model are presented.

The Lattice Structure

The filter structure known as lattice is a recursive structure that presents some interesting properties. One of its most important properties, which will be of special interest here, is its capacity for representing transmission-reflection phenomena in a very simple way. In fact, it is for this reason that lattice structures have been so popular in speech synthesis (Gray & Markel, 1973). In the problem of seismic exploration, a very similar approach can be used.

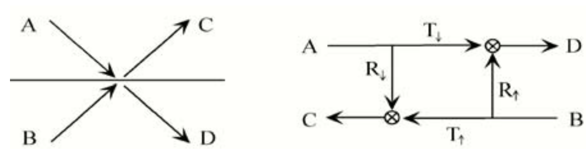

FIG. 1. Transmission-Reflection at an Interface Between two Media

Consider the situation presented in figure 1, in which an interface between two media and four wave rays are illustrated. As can be seen from the figure, rays A and B represent all the rays impinging on the interface from above and below respectively; and rays C and D represent all the rays emerging from the interface to the upper and lower media respectively. The structure depicted in the right hand side of figure 1 constitutes a flow diagram of the transmission-reflection process that takes place in the considered interface. The values of $T_\downarrow$, $T_\uparrow$, $R_\downarrow$ and $R_\uparrow$ represent the transmission and reflection coefficients in the upward and downward directions. Notice how the structure combines the transmitted and reflected rays into the emerging rays in such a way that ray C contains both the reflected energy from ray A and the transmitted energy from ray B; similarly, ray

D contains the reflected energy from B and the transmitted energy from A.

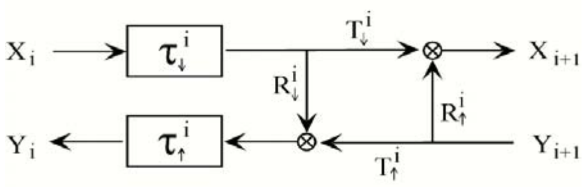

FIG. 2. Normalized Lattice Section for Earthen Formation Modeling

By including two delay operators into the structure shown in figure 1, and introducing some notation changes, the one presented in figure 2 is obtained. This type of structure is known as a normalized lattice section (Oppenheim & Schafer, 1975). In it, the delay operators $\tau_{\downarrow}^{i}$ and $\tau_{\uparrow}^{i}$ represent the travel times for the down-going ray X_i and the up-going ray Y_i in medium i, which is the one above the interface under consideration. On the other hand, rays X_{i+1} and Y_{i+1} denote the down-going and up-going rays in medium $i+1$, which is the one below the interface under consideration.

The outputs, Y_i and X_{i+1}, of the normalized lattice section presented in figure 2 are related to its inputs, X_i and Y_{i+1}, by means of the following difference equations:

$$X_{i+1}(t) = T_{\downarrow}^{i}\, X_i(t-\tau_{\downarrow}^{i}) + R_{\uparrow}^{i}\, Y_{i+1}(t)\,, \qquad (1)$$

$$Y_i(t+\tau_{\uparrow}^{i}) = R_{\downarrow}^{i}\, X_i(t-\tau_{\downarrow}^{i}) + T_{\uparrow}^{i}\, Y_{i+1}(t)\,. \qquad (2)$$

Notice that a geological model of N layers can be constructed by interconnecting $N-1$ sections like the one presented in figure 2. The parameters of such a model would be the delays and the transmission and reflection coefficients of each of the $N-1$ sections considered.

Important Considerations

Although the *IIR* convolutional model described here presents some advantages over the traditional *FIR* model, it still constitutes a gross approximation of the physical phenomenon and presents some limitations too. The use of the *FIR* convolutional model involves three important assumptions about the problem. First, it is assumed that the seismic wavelet is known; second, the source-receiver offset is zero and, third, the wave energy remains constant along the whole down-going path. In the case of the *IIR* convolutional model, the third of the mentioned assumptions does not represents a limitation any more. This is because the new model incorporates the effects of the transmission coefficients at each interface.

On the other hand, the assumptions of known wavelet and zero offset are still present in the *IIR* convolutional model. While the known wavelet restriction can be dealt with by using conventional wavelet estimation techniques (Robinson & Treitel, 1980), the zero offset assumption* imposes some particular conditions on the parameters of each lattice section. For the particular case of compressional waves (P-P), the reflection and transmission coefficients of the ith lattice section can be expressed in terms of the single reflection coefficient $R_{\downarrow}^{i}$ (Waters, 1981), which is going to be denoted as R_i in order to simplify the notation. In this manner:

$$R_{\downarrow}^{i} = R_i\,, \qquad (3)$$

$$T_{\downarrow}^{i} = 1 - R_i\,, \qquad (4)$$

$$R_{\uparrow}^{i} = -R_i\,, \qquad (5)$$

$$T_{\uparrow}^{i} = 1 + R_i\,. \qquad (6)$$

With respect to the delay terms, it can be said that under the assumptions of normal incidence, $\tau_{\downarrow}^{i}$ and $\tau_{\uparrow}^{i}$ must be equal†. Again, in order to simplify the notation, the common delay term of the ith lattice section will be denoted as τ_i. In this way:

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*Which in a flat layered model is totally equivalent to normal incidence.

†Notice that in normal incidence, there is no mode conversion.

$$\tau_{\downarrow}^{i} = \tau_{\uparrow}^{i} = \tau_i \,. \tag{7}$$

It can be noticed from equations (3), (4), (5), (6) and (7) that the number of unknown parameters in the model has been substantially reduced. Only two parameters per lattice section, R_i and τ_i, are required. In this way, for an earthen formation model of N layers, a total of $2\,(N-1)$ parameters must be determined.

The *IIR* convolutional model provides a more realistic representation of the physical problem than the *FIR* model because it introduces two new elements in the synthetic traces. First, it includes the attenuation due to the transmission coefficients; second, it includes the effects of multiple reflections. However, the price to pay for these improvements is the non linearity of the inverse problem formulation.

Synthetic Traces

After incorporating the results obtained in (3), (4), (5), (6) and (7) into the lattice section illustrated in figure 2, the structure presented in figure 3 is obtained. Additionally, initial and terminal sections, for representing the earth surface and the deepest layer, are illustrated.

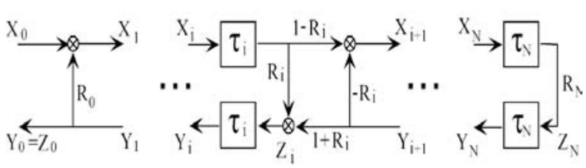

FIG. 3. Lattice Sections for the Generation of Synthetic Traces

Notice from figure 3, that the intermediate variable Z_i have been defined. In this way, Y_i can be expressed in terms of Z_i as follows:

$$Y_i(t) = Z_i(t-\tau_i)\,. \tag{8}$$

The use of Z_i instead of Y_i simplifies the computational implementation of the lattice structure. By replacing (8), as well as (3), (4), (5), (6) and (7), into (1) and (2) the following difference equations are obtained:

$$X_{i+1}(t) = \begin{array}{l}(1-R_i)\,X_i(t-\tau_i) \\ -R_i\,Z_{i+1}(t-\tau_{i+1})\,,\end{array} \tag{9}$$

$$Z_i(t) = \begin{array}{l}R_i\,X_i(t-\tau_i) \\ +(1+R_i)\,Z_{i+1}(t-\tau_{i+1})\,.\end{array} \tag{10}$$

Equations (9) and (10) describe the input-output relationships for the inner lattice sections ($i = 1, 2, \ldots N-1$) illustrated in figure 3. For the initial ($i=0$) and final ($i=N$) sections, the following equations must be considered:

$$X_1(t) = X_0(t) + R_0\,Z_1(t-\tau_1)\,, \tag{11}$$

$$Z_0(t) = Z_1(t-\tau_1)\,, \tag{12}$$

$$Z_N(t) = R_N\,X_N(t-\tau_N)\,, \tag{13}$$

where N is the total number of layers in the model, $X_0(t)$ is the seismic wavelet and the most typical settings for R_0, R_N and τ_N are: $R_0 = 1$ and $R_N = \tau_N = 0$.

The computation of a synthetic trace can be performed by the recursive use of (9), (10), (11), (12) and (13). Figure 4 shows two synthetic traces; one was computed with the conventional *FIR* convolutional model and the other with the proposed *IIR* model.

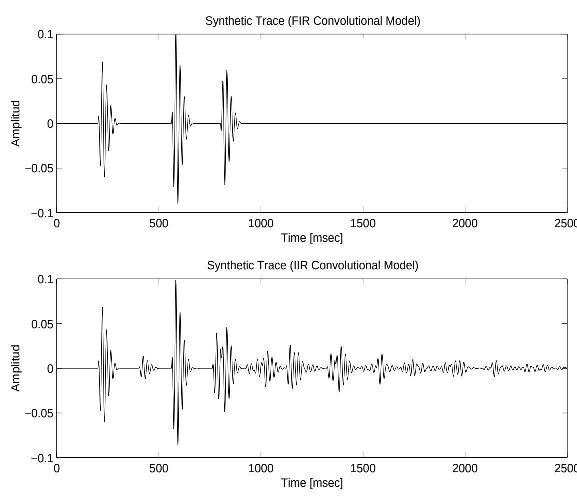

FIG. 4. Synthetic Seismic Traces

Both traces in figure 4 correspond to a model of four layers with $R_1 = 0.2$, $R_2 = 0.3$ and $R_3 = -0.2$; and $\tau_1 = 100$, $\tau_2 = 180$ and $\tau_3 = 120$.

Objective Functions

An important property of the convolutional model is the independence between its two parameter sets. It can be easily seen that in the model space, time delays and reflection coefficients are independent of each other[‡]. In the data space, on the other hand, the independence between the two parameter sets is not preserved. This is due to the band-limited character of the seismic wavelet. However, for practical purposes, some degree of independence can still be assumed by noticing that each of the parameter sets produce different effects on the resulting seismic data. In fact, while delay terms determine the delay of the events in a seismic trace, reflection coefficients determine the relative amplitude of such events.

The previous discussion suggests that optimization of each parameter set can be performed separately. However, since independence is definitively not an issue in the data space under consideration, an iterative procedure will be needed in order to reach optimal sets in both of the model subspaces. Additionally, as will be seen next, the definition of specific objective functions for each parameter set is recommended in order to increase the success rate of the inversion process.

The most natural and simple idea about an objective function follows from the concept of the squared error. In this way, a possible objective function can be defined as:

$$E_{trace} = \sum_{n=1}^{nsamples} \left(T_r[n] - T_s[n]\right)^2 , \qquad (14)$$

where T_r is the real seismic trace, T_s is the synthetic seismic trace, and *nsamples* is the total number of samples available.

[‡]The conditional probability of one parameter set occurrence is independent of the occurrence of the other parameter set.

The error function defined in (14) is a function of the earthen model parameters; and it has its global minimum at that point of the model space that corresponds to the correct earthen formation. However, it also has several local minima which represent suboptimal models. This problem is illustrated in figure 5 for a very simple model of only two layers with $R_1 = 0.2$ and $\tau_1 = 70$.

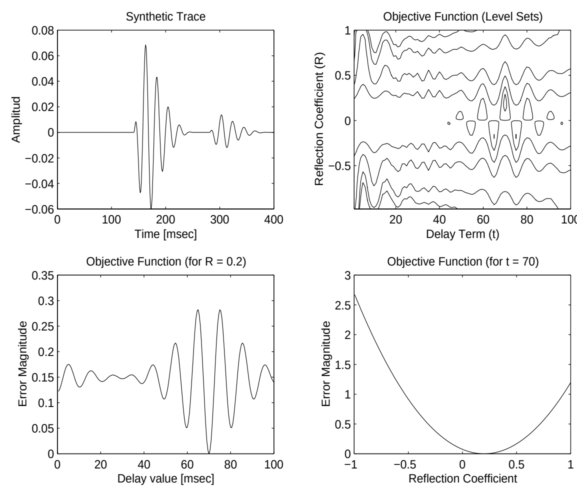

FIG. 5. Trace's Squared Error Objective Function

Notice how, even for a very simple example like the one presented in figure 5, the complexity of the inverse problem is considerable. However, it can also be noticed that the problem of multiple local minima primarily affects the delay parameters.

The results depicted in figure 5, suggest that the objective function defined in (14) is not the most appropriate for inverse modeling, at least for the case of the delay terms. The problem of multiple minima, can be reduced by considering the envelope of the seismic trace instead of the seismic trace itself. Based on this idea, a new objective function can be defined as:

$$\begin{aligned} E_{env} = & \sum_{n=1}^{nsamples} \left[\left(\sum_k |T_r[k]|\, F[n-k]\right)\right. \\ & \left. - \left(\sum_k |T_s[k]|\, F[n-k]\right)\right]^2 , \end{aligned} \qquad (15)$$

where $||$ is the absolute value operator and $F[n]$ is a low pass filter with cutoff frequency close to the seismic wavelet's central frequency. Basically, what (15) does is to compute the squared error of the traces' envelopes.

The new objective function E_{env} is very suitable for the determination of the delay terms, but very deficient for computing the reflection coefficients. This is because when the absolute value operator is applied in order to obtain the envelopes, the information related to the the reflection coefficient's sign is lost. A third objective function, which preserves the reflection coefficient sign information can be defined as follows:

$$E_{lp} = \sum_{n=1}^{nsamples} \left[\left(\sum_k T_r[k]\, F[n-k] \right) - \left(\sum_k T_s[k]\, F[n-k] \right) \right]^2 , \qquad (16)$$

where the absolute value operator has been dropped. Basically, what (16) does is to compute the squared error of the low pass filtered traces.

Figure 6 illustrates an example of the trace envelope and the low pass filtered trace used for computing the objective functions (15) and (16).

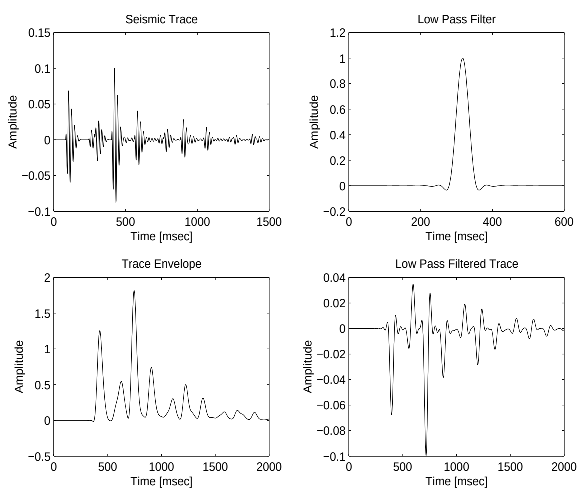

FIG. 6. Trace Envelope and Low Pass Filtered Trace

Notice from the figure, how the shape of the trace envelope still suggests the existent of local minima. However, it can be proved that their number has been substantially reduced. In fact, observe from figure 5 that local minima in the original error function (14) are related to phase effects on the seismic trace due to small biases of the delay terms. On the other hand, according to figure 6, local minima in the envelope objective function (15) would be related to large deviations of the delay terms resulting from erroneous matching between different events in the seismic trace. This fact represents the main reason for considering the inclusion of a non linear inversion technique such as genetic algorithms.

Search Methods

A hybrid optimization scheme will be used for the inversion problem studied here since such schemes have proved to provide good results in geophysical inversion problems (Chunduru, 1996). In this kind of schemes, the search starts with a global search algorithm, which is used to find a 'good region' in the model space. Once this has been done, the local algorithm continues the search for the minima, which generally achieves it faster than the global method. In this way, hybrid optimization combines the advantages of both global and local search, robustness and speed, respectively.

The global search method to be used in the presented problem is the genetic algorithm. It was selected due to its high performance and robustness in problems characterized by local minima multiplicity (Goldberg, 1989). Genetic algorithms are based on the analogy between the way biological communities evolve and the problem of maximizing a function of multiple variables. They perform their search by considering a 'population' of models instead of a single model at a time.

The local search method to be used is the one due to Nelder and Mead (1965). This method is based on a direct search simplex method. In this kind of algorithms, a simplex is constructed in the model space and a new point is evaluated at each iteration. When the point evaluated is better than one of the vertices of the simplex, the current simplex is updated by replacing the vertex with the new point. This process is repeated until the size of the simplex has been reduced up to a predefined tolerance value. This particular kind of local search methods do not use information of the local derivatives.

The Inversion Algorithm

Based on the previous discussions the definitive inversion algorithm can be designed. Two features must be incorporated into the method; first, optimization of both parameter sets (delays and reflection coefficients) should be performed separately, and second, the first global optimization must be performed on the delay subspace since the problem of multiple local minima affects mainly the time delay terms.

In this way, the inversion method can be described as follows:

- Global search: It defines an initial guess for the reflection coefficients and uses genetic algorithms for finding a 'good region' in the delay subspace. The objective function (15) must be used.
- Local search: It uses an initial model consisting of the initial reflection coefficient guess and the delay set obtained in the global search. The local search is performed in two steps that are repated iteratively until convergence is achieved. These two steps are:
  - Reflection coefficient search: It keeps the delay values fixed, and updates the reflection coefficients by searching in the reflection coefficient subspace. The objective function (16) must be used.
  - Time delay search: It keeps the reflection coefficients fixed and searches in the delay subspace. The objective function (15) must be used.

Additionally, if a good representation of the seismic wavelet is available, it is possible two finish the local search iterative procedure by using the trace squared error (14). In this way, the use of different objective functions during the search leads the algorithm from an initial 'gross' adjustment to a final 'fine' adjustment of the parameters.

Simulation Examples

In this section, two simulation examples are presented in order to illustrate the methodology developed in the previous sections.

SIMULATION 1*: Typical Formation* In this simulation, a formation model of four layers was considered. The parameters of the model were defined as follows; delay terms: $\tau_1 = 115$, $\tau_2 = 50$ and $\tau_3 = 75$; reflection coefficients: $R_1 = 0.20$, $R_2 = 0.18$ and $R_3 = -0.24$.

For inverse modeling, a four layer formation model was considered too. The synthetic seismic data and the obtained results are presented in figure 7. The delay values obtained during the global search procedure were $\tau_1 = 133$, $\tau_2 = 60$ and $\tau_3 = 62$; which were finally adjusted, along with the reflection coefficients, by the local search algorithm. The parameters of the resulting model were: $\tau_1 = 115$, $\tau_2 = 50$, $\tau_3 = 75$, $R_1 = 0.2000$, $R_2 = 0.1801$ and $R_3 = -0.2399$.

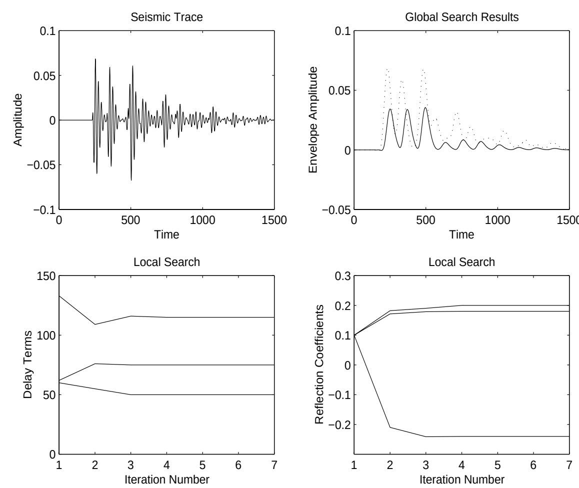

FIG. 7. Seismic Inversion in a Typical Formation

SIMULATION 2*: Marine Environment* In this simulation, a formation model of three layers was considered. The parameters of the model were defined as follows: delay terms, $\tau_1 = 70$ and $\tau_2 = 160$, and reflection coefficients, $R_1 = 0.90$ and $R_2 = -0.20$.

The synthetic seismic data and the obtained results are presented in figure 8. In this example, the inversion was performed in two steps. This was done in order to handle the problem of the high-energy multiple reflections in a better way. First, the reflection coefficient responsible for the reflections was estimated by considering a model of just two layers. The inversion algorithm previously described was applied to the proposed model and the following model parameters were obtained: $\tau = 70$ and $R = 0.8907$. The results of the global search algorithm corresponding to this first step is shown in the upper right plot of figure 8. The delay value obtained by the global search was $\tau = 69$.

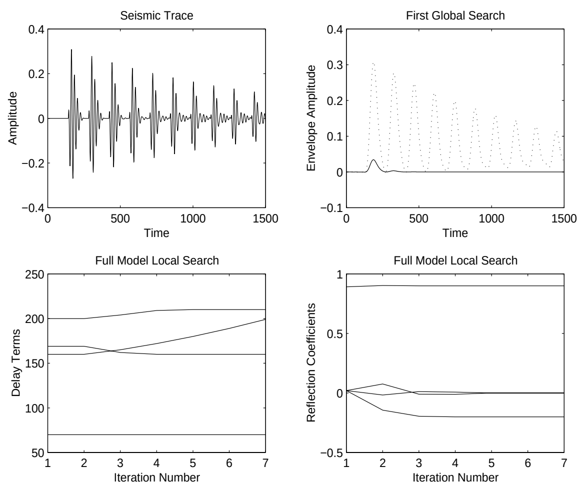

FIG. 8. Seismic Inversion in a Marine Environment

In the second step, a five layer formation model was considered. The inversion algorithm previously described was applied again, with the difference that global search was performed only on the subspace defined by τ_2, τ_3 and τ_4; i.e. the previously obtained value of $\tau_1 = 70$ was considered a fixed parameter. The reflection coefficient set used was: $R_1 = 0.8907$ and $R_2 = R_3 = R_4 = 0.1\,(1 - R_1^2)$. The delay terms resulting from this second global search were: $\tau_2 = 169$, $\tau_3 = 160$ and $\tau_4 = 200$.

Finally, all the model parameters were adjusted by the local search algorithm. The parameters of the resulting model were: $\tau_1 = 70$, $\tau_2 = 160$, $\tau_3 = 199$, $\tau_4 = 210$; and $R_1 = 0.9000$, $R_2 = -0.1999$, $R_3 = 2.305\,10^{-5}$, $R_4 = -3.761\,10^{-5}$. This second local search is illustrated in the lower half of figure 8.

Conclusions

In the procedure developed in this paper, two important issues must be emphasized. First, the introduction of an *IIR* convolutional model for representing the subsurface allows the inclusion of both transmission effects and multiple reflections into the computed synthetic traces. As was shown by the second example of the previous section, this can constitute a powerful tool for the analysis of seismic data in which multiple reflections mask the events of interest. Such a situation is very common within the seismic data recorded in marine and other aquatic environments. Of particular concern, it is the case of Lake Maracaibo in Venezuela (Sixma, 1996).

The second important issue is the fact that optimization is performed separately on the delay and reflection coefficient subspaces. As shown, the error surface complexity and the problem of minima multiplicity affect the delay parameters much more than they affect the reflection coefficients. It can be verified, with simulation experiments, that the initial global search performed on the delay subspace plays a crucial role in the success of the whole inversion procedure.

The effects of noise on the stability of the proposed algorithm have not been studied yet.

References

Chunduru, R. 1996, *Global and Hybrid Optimization in Geophysical Inversion*, Doctoral Dissertation, The University of Texas at Austin.

Gray, A., and Markel, J. 1973, *Digital Lattice and Ladder Filter Synthesis*, IEEE Trans. Audio Electroacoustics AU-21, 491-500.

Goldberg, D. 1989, *Genetic Algorithms*, Addison Wesley.

Nelder, J., and Mead, R. 1965, *A Simplex Method for Function Minimization*, Computer J., vol 7, 308-313.

Oppenheim, A., and Schafer, R. 1975, *Digital Signal Processing*, Prentice Hall.

Robinson, E., and Treitel, S. 1980, *Geophysical Signal Analysis*, Prentice Hall.

Sixma, E. 1996, *Guided Waves in Lake Maracaibo*, SOVG, 8vo Congreso Venezolano de Geofísica, 262-270.

Waters, K. 1981, *Reflection Seismology*, John Wiley & Sons.